Unit 4 Study Guide

1. $\frac{dy}{dx} = 2\sqrt{x} \quad (4,12)$

$y = \int 2\sqrt{x}\,dx = 2\int x^{1/2}\,dx$

$y = \frac{2x^{3/2}}{3/2} + C = \frac{4}{3}x^{3/2} + C$

$12 = \frac{4}{3}(4)^{3/2} + C$

$12 = \frac{32}{3} + C$

$C = 4/3$

$\boxed{y = \frac{4}{3}x^{3/2} + \frac{4}{3}}$

2. a. $\int (5-x)\,dx$

$5x - \frac{x^2}{2} + C$

$\boxed{5x - \frac{1}{2}x^2 + C}$

b. $\int \left(\frac{x^2+x+1}{\sqrt{x}}\right)dx$

$\int \left(\frac{x^2}{x^{1/2}} + \frac{x}{x^{1/2}} + \frac{1}{x^{1/2}}\right)dx$

$\int (x^{3/2} + x^{1/2} + x^{-1/2})\,dx$

$\frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$

$\boxed{\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C}$

c. $\int \left(\frac{\cos x}{1-\cos^2 x}\right)dx$

$\int \frac{\cos x}{\sin^2 x}\,dx$

$\int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx$

$\int \cot x \csc x\,dx$

$\boxed{-\csc x + C}$

3. $\sum_{k=3}^{9} 2k-5 = \boxed{49}$

$(2\cdot3-5)+(2\cdot4-5)+(2\cdot5-5)+$
$(2\cdot6-5)+(2\cdot7-5)+(2\cdot8-5)+(2\cdot9-5)$

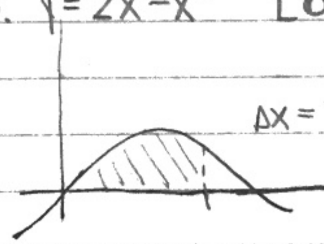
4. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4i^2(i-1)}{n^4} = \lim_{n\to\infty} \frac{4}{n^4} \sum_{i=1}^{n} i^3 - i^2$

$\lim_{n\to\infty} \frac{4}{n^4}\left[\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}\right]$

$\lim_{n\to\infty} \frac{4n^2(n^2+1)^2}{4n^4} + \frac{4n(n+1)(2n+1)}{6n^4}$

$1 + 0 = \boxed{1}$

5. $y = 2x - x^3 \quad [0,1]$



$\Delta x = \frac{1}{n} \quad c_i = 0 + \frac{i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\frac{1}{n} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} 2\left(\frac{i}{n}\right) - \left(\frac{i}{n}\right)^3$

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{2i}{n} - \frac{i^3}{n^3}$

$\lim_{n\to\infty} \frac{1}{n}\left[\frac{2n(n+1)}{2n} - \frac{n^2(n+1)^2}{4n^3}\right]$

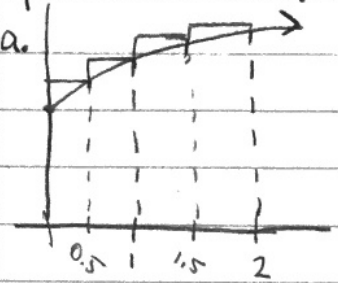
$\lim_{n\to\infty} \frac{1}{n}\left[n+1 - \frac{(n+1)^2}{4n}\right]$

$\lim_{n\to\infty} \frac{n}{n} + \frac{1}{n} - \frac{(n+1)^2}{4n^2}$

$\lim_{n\to\infty} 1 + \frac{1}{n} - \frac{(n+1)^2}{4n^2} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

6. $y=\sqrt{x}+2 \quad [0,2]$

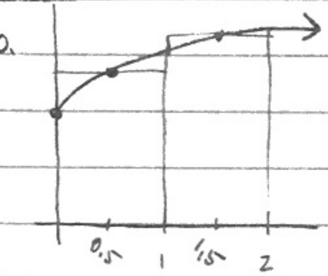
a.



$A=\frac{1}{2}f(0.5)+\frac{1}{2}f(1)+\frac{1}{2}f(1.5)+\frac{1}{2}f(2)$

$\boxed{A=6.173}$

b.



$A=(1)f(0.5)+1f(1.5)$

$\boxed{A=5.932}$

7. $\int_1^3 3x^2\,dx=\lim_{n\to\infty}\sum_{i=1}^{n} f\left(1+\frac{2i}{n}\right)\frac{2}{n}$

$\Delta x=\frac{2}{n}$

$c_i=1+\frac{2i}{n}$

8. $\int_0^3 f(x)\,dx=4 \quad \int_3^6 f(x)\,dx=-1$

a. $\int_0^6 f(x)\,dx=\int_0^3 f(x)\,dx+\int_3^6 f(x)\,dx$

$4\ -1=\boxed{3}$

b. $\int_6^3 f(x)\,dx=-\int_3^6 f(x)\,dx=\boxed{1}$

c. $\int_3^3 f(x)\,dx=\boxed{0}$

d. $\int_3^6 -5f(x)\,dx=-5\int_3^6 f(x)\,dx$

$-5(-1)=\boxed{5}$

9. $\int_1^8 \sqrt{\frac{2}{x}}\,dx=\int_1^8 \frac{\sqrt{2}}{\sqrt{x}}\,dx=\sqrt{2}\int_1^8 \frac{1}{x^{1/2}}\,dx=$

$\sqrt{2}\int_1^8 x^{-1/2}\,dx=\sqrt{2}\left[2x^{1/2}\right]_1^8$

$\sqrt{2}\left[2\sqrt{8}-2\sqrt{1}\right]=2\sqrt{16}-2\sqrt{2}$

$\boxed{8-2\sqrt{2}}$

10. $f(x)=\frac{5}{x^3} \quad [2,6]$

$\int_2^6 \frac{5}{x^3}\,dx=f(c)\,(6-2)$

$5\int_2^6 x^{-3}\,dx=f(c)\cdot 4=\frac{5}{4}\int_2^6 x^{-3}\,dx=f(c)$

$\frac{5}{4}\left[\frac{x^{-2}}{-2}\right]_2^6=f(c) \Rightarrow \frac{5}{4}\left[\frac{-1}{2x^2}\right]_2^6=f(c)$

$\frac{5}{4}\left[\frac{-1}{2\cdot 6^2}-\frac{-1}{2\cdot 2^2}\right]=f(c)$

$\frac{5}{36}=f(c)$

$\frac{5}{36}=\frac{5}{x^3}$

$5x^3=5\cdot 36$

$x^3=36$

$\boxed{x=\sqrt[3]{36}\approx 3.302}$

11. $f(x) = \cos x \quad [0, \pi/2]$

Average Value

$\frac{1}{b-a} \int_a^b f(x)\,dx$

$\frac{1}{\frac{\pi}{2}-0} \int_0^{\pi/2} \cos x\,dx$

$\frac{2}{\pi} \int_0^{\pi/2} \cos x\,dx$

$\frac{2}{\pi} [\sin x]_0^{\pi/2}$

$\frac{2}{\pi} [\cdot \sin(\pi/2) - \sin(0)]$

$\frac{2}{\pi} [1 - 0] \; \boxed{2/\pi}$

b. $\int x^2 \sqrt{x^3+2}\,dx \quad u = x^3+2$

$du = 3x^2\,dx$

$\frac{1}{3} \int 3x^2 \sqrt{x^3+2}\,dx$

$\frac{1}{3} \int \sqrt{u}\,du = \frac{1}{3} \int u^{1/2}\,du$

$\frac{1}{3} \frac{u^{3/2}}{3/2} + C = \frac{2}{9}(u)^{3/2} + C$

$\boxed{\frac{2}{9}(x^3+2)^{3/2} + C}$

c. $\int (x+1)\sqrt{2-x}\,dx \quad u = 2-x$

$\int (2-u+1)\sqrt{u}\,(-du) \quad du = -dx$

$-\int (3-u)(u)^{1/2}\,du \quad x = 2-u$

$-\int (3u^{1/2} - u^{3/2})\,du$

$-\left[\frac{3u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right] + C$

$-2u^{3/2} + \frac{2}{5}u^{5/2} + C$

$\boxed{-2(2-x)^{3/2} + \frac{2}{5}(2-x)^{5/2} + C}$

12. a. $F(x) = \int_4^x \sqrt{t}\,dt$

$\boxed{F'(x) = \sqrt{x}}$

b. $F(x) = \int_4^{2x} (t^2+4)\,d$

$F'(x) = [(2x)^2+4] \cdot 2$

$F'(x) = 2(4x^2+4)$

$\boxed{F'(x) = 8x^2+8}$

d. $\int \frac{\cos^3\theta}{2-2\sin^2\theta}\,d\theta = \frac{1}{2}\int \frac{\cos^3\theta}{1-\sin^2\theta}\,d\theta$

$\frac{1}{2}\int \frac{\cos^3\theta}{\cos^2\theta}\,d\theta$

$\frac{1}{2}\int \cos\theta\,d\theta = \boxed{\frac{1}{2}\sin\theta + C}$

13. a. $\int \sqrt[3]{1-2x^2}\,(4x)\,dx \quad u = 1-2x^2$

$du = -4x\,dx$

$-\int \sqrt[3]{1-2x^2}\,(-4x)\,dx$

$-\int \sqrt[3]{u}\,du$

$-\int u^{1/3}\,du$

$-\frac{u^{4/3}}{4/3} + C = -\frac{3}{4}u^{4/3} + C$

$\boxed{-\frac{3}{4}(1-2x^2)^{4/3} + C}$

14. a. $\int_{-2}^{4} x^2(x^3+8)^2\,dx$ $u=x^3+8$

$du=3x^2dx$

If $x=-2,\ u=0$

$x=4,\ u=72$

$\frac{1}{3}\int_{-2}^{4} 3x^2(x^3+8)^2dx$

$\frac{1}{3}\int_{0}^{72} u^2\,du = \frac{\frac{1}{3}u^3}{3}\Big]_0^{72}$

$\frac{1}{9}u^3\Big]_0^{72} = \frac{1}{9}\left[72^3-0^3\right]$

$\boxed{41,472}$

b. $\int_{\pi/3}^{\pi/2}(x+\cos x)\,dx$

$\frac{x^2}{2}+\sin x\Big]_{\pi/3}^{\pi/2}$

$\left(\frac{(\pi/2)^2}{2}+\sin(\pi/2)\right)-\left(\frac{(\pi/3)^2}{2}+\sin(\pi/3)\right)$

2.23370055 − 1.414336759

$\boxed{0.819}$

16. $v(t)=4-t^2$ [1,4]

Avg of $v(t)$ =

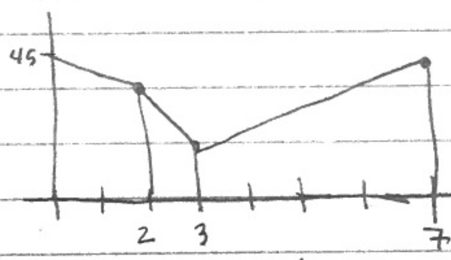
$\frac{1}{4-1}\int_1^4(4-t^2)\,dt$

$\frac{1}{3}\int_1^4(4-t^2)\,dt = \frac{1}{3}\left[4t-\frac{t^3}{3}\right]_1^4$

$\frac{1}{3}\left[\left(4\cdot4-\frac{4^4}{3}\right)-\left(4\cdot1-\frac{1^3}{3}\right)\right]$

$\boxed{-3}$

17. $\int_0^7 R(t)\,dt \approx \boxed{186.5}$



$\frac{1}{2}(2)(45+29)+\frac{1}{2}(1)(29+12)+\frac{1}{2}(4)(12+34)$

$\boxed{186.5}$

15. $\frac{dy}{dx}=\frac{-48}{(3x+5)^3}$ $(-1,3)$

$y=\int -48(3x+5)^3\,dx$

$y=-48\int(3x+5)^{-3}\,dx$ $u=3x+5$

$du=3dx$

$y=-48\cdot\frac{1}{3}\int 3(3x+5)^{-3}\,dx$

$y=-16\int u^{-3}\,du$

$y=-16\cdot\frac{u^{-2}}{-2}+C$

$y=\frac{8}{u^2}+C \Rightarrow y=\frac{8}{(3x+5)^2}+C$

$3=\frac{8}{(3\cdot-1+5)^2}+C \Rightarrow 3=\frac{1}{2}+C$

$C=5/2$

$\boxed{y=\frac{8}{(3x+5)^2}+\frac{5}{2}}$

18. $a(t)=8t$ $v(0)=10$ $s(0)=7$

$v(t)=\int a(t)\,dt$

$v(t)=\frac{8t^2}{2}+C$

$v(t)=4t^2+C$

$v(0)=10=4(0)^2+C$

$10=C$

$v(t)=4t^2+10$

$s(t)=\int v(t)\,dt$

$s(t)=\int(4t^2+10)\,dt$

$s(t)=\frac{4t^3}{3}+10t+C$

$s(t)=\frac{4}{3}t^3+10t+C$

$7=\frac{4}{3}(0)^3+10(0)+C$

$7=C$

$s(t)=\frac{4}{3}t^3+10t+7$

$s(4)=\frac{4}{3}(4)^3+10(4)+7$

$\boxed{s(4)=132.333}$

19. $s(t) = 40 + 12\sin(0.2t)$

$$\int_0^4 s(t)\,dt = \boxed{178.198 \text{ tons}}$$