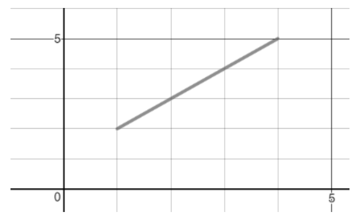
Unit 3 – Applications of Differentiation – Study Guide

3.1 Extrema on an Interval

1. Find the critical numbers of the function:
   a. $f(x) = x^2(2x^2 - 32)$
   b. $f(x) = \frac{4x}{x^2+1}$
2. Locate the absolute extrema of the function on the closed interval:
   a. $f(x) = x^2 + 2x - 4$ on $[-2,1]$
   b. $f(x) = x - 2 - \cos x$ on $[0, 2\pi]$
3. Locate the absolute extrema of the function $f(x) = x + 1$ on the indicated intervals:
   a. [1, 4]
   b. (1, 4]
   c. [1, 4)
   d. (1, 4)



3.2 Rolle's Theorem and the Mean Value Theorem

4. Determine whether Rolle's Theorem can be applied to *f* on the given interval. If Rolle's Theorem can be applied, find all values c such that $f'(c) = 0$.
   a. $f(x) = x^2 - 5x + 4$, [1,4]
   b. $f(x) = \frac{x^2-1}{x}$, [-1,1]
5. Determine whether the Mean Value Theorem can be applied to *f* on the given interval [*a, b*]. If the MVT can be applied, find all values of c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
   a. $f(x) = x(x^2 - x - 2), [-1,1]$
   b. $f(x) = \frac{x+1}{x}$, $[\frac{1}{2}, 2]$

3.3 Increasing and Decreasing Functions and the First Derivative Test

6. Determine the open intervals on which the function given is increasing and decreasing. Justify your answer.
   a. $f(x) = x^3 - 6x^2 + 15$
   b. $f(x) = \frac{x+3}{x^2}$
7. Use the first derivative test to determine any relative extrema on the given function. Justify your answer.
   a. $f(x) = x^{2/3} - 4$
   b. $f(x) = x^4 - 6x^2$

3.4 Concavity and the Second Derivative Test

8. Find all points of inflection and the intervals of concavity of the graph of the function.
   a. $f(x) = 2x^3 - 3x^2 - 12x + 5$
   b. $f(x) = x\sqrt{x+1}$
9. Find all relative extrema. Use the 2nd Derivative test where applicable.
   a. $f(x) = -(x-5)^2$
   b. $f(x) = x^3 - 9x^2 + 27x$
10. Sketch a function f having the given characteristics.
   a. $f(0) = f(2) = 0$,
      $f'(x) < 0$ if $x < 1$,
      $f'(x) > 0$ if $x > 1$,
      $f'(1) = 0$,
      $f''(x) > 0$

3.5 Limits At Infinity

11. Find the limits of each. If the limit does not exist, specify $\pm\infty$.
    a. $\lim_{x\to\infty} \frac{3-2x}{3x^3-1}$
    b. $\lim_{x\to\infty} \frac{3-2x}{3x-1}$
    c. $\lim_{x\to\infty} \frac{3-2x^2}{2x-1}$

3.6 Curve Sketching

12. Sketch the graph of the equation. Look for extrema, intercepts, points of inflection, asymptotes and concavity.
    a. $f(x) = \frac{x^2+1}{x^2-9}$
13. Let f be a function that is continuous on the interval $[-1,4)$. The function is twice differentiable except at $x = 0$. Use the table provided below to answer questions about $f(x)$.

x	-1	$-1 < x < 0$	0	$0 < x < 2$	2	$2 < x < 3$	3	$3 < x < 5$
$f(x)$	1	Positive	0	Negative	-2	Negative	0	Positive
$f'(x)$	-3	Negative	0	Negative	DNE	Positive	4	Positive
$f''(x)$	2	Positive	0	Negative	DNE	Positive	0	Negative

a. For $-1 < x < 4$, find all values of *x* at which f has a relative extremum. Determine whether *f* has a relative maximum or relative minimum at each of these values. Justify your answer.
b. Sketch the graph of the function f described in the table above, making sure it has all the characteristics described in the table.

3.7 Optimization Problems

14. Find two positive numbers such that the second number is the reciprocal of the first and the sum is a minimum.
15. Find the point on the graph of $f(x) = x^2$ that is closest to the point $(4,0)$.
16. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should he use so that the enclosed area will be a maximum?

3.9 Linearization

17. Given $f(x) = x^5$, find the linearization of the function at $x = 2$. Then use the linearization to approximate $f(2.1)$.