# ★ Derivatives Review Worksheet ★

1. $f(x) = \frac{3}{x}$ $(3,1)$

$\lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{f(3+\Delta x) - 1}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\frac{3}{3+\Delta x} - 1}{\Delta x} \cdot \frac{3+\Delta x}{3+\Delta x}$

$\lim_{\Delta x \to 0} \frac{3 - (3+\Delta x)}{\frac{3+\Delta x}{\Delta x}}$

$\lim_{\Delta x \to 0} \frac{-\Delta x}{\frac{3+\Delta x}{\Delta x}}$

$\lim_{\Delta x \to 0} \frac{-\Delta x}{3+\Delta x} \cdot \frac{1}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{-1}{3+\Delta x} = \frac{-1}{3+0} = \boxed{\frac{-1}{3}}$

2. $f(x) = \sqrt{x} + 1$

$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} + 1 - (\sqrt{x}+1)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$

$\lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})}$

$\lim_{\Delta x \to 0} \frac{\Delta x \, 1}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})}$

$\lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$

3. $h(x) = f(x) \cdot g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(-8) = f(-8)g'(-8) + g(-8)f'(-8)$

$h'(-8) = (2)(-3) + (4)(-1)$

$h'(-8) = -6 - 4 = \boxed{-10}$

4. $h(x) = \frac{f(x)}{g(x)}$

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$h'(4) = \frac{g(4)f'(4) - f(4)g'(4)}{[g(4)]^2}$

$h'(4) = \frac{(7)(5) - (1)(2)}{7^2} = \frac{35-2}{49} = \boxed{\frac{33}{49}}$

5. $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(-8) = f'(g(-8)) \cdot g'(-8)$

$h'(-8) = f'(4) \cdot -3$

$h'(-8) = 5 \cdot -3 = \boxed{-15}$

6. $y = \frac{-3}{x^4} = -3x^{-4}$

$y' = 12x^{-5}$ $\boxed{y' = \frac{12}{x^5}}$

7. $y = x^2(3x+1)$

$y' = x^2(3) + (3x+1)(2x)$

$y' = 3x^2 + 6x^2 + 2x$

$\boxed{y' = 9x^2 + 2x}$

8. $y = \frac{4x+1}{x^3-2}$

$y' = \frac{(x^3-2)(4)-(4x+1)(3x^2)}{(x^3-2)^2}$

$y' = \frac{4x^3-8-12x^3-3x^2}{(x^3-2)^2}$

$\boxed{y' = \frac{-8x^3-3x^2-8}{(x^3-2)^2}}$

9. $y = (3x^4-5x)^4$

$y' = 4(3x^4-5x)^3 \cdot (12x^3-5)$

$\boxed{y' = 4(12x^3-5)(3x^4-5x)^3}$

10. $y = \frac{(x^2+5)^3}{2x-1}$

$y' = \frac{(2x-1)\cdot 3(x^2+5)^2(2x) - (x^2+5)^3(2)}{(2x-1)^2}$

$y' = \frac{6x(2x-1)(x^2+5)^2 - 2(x^2+5)^3}{(2x-1)^2}$

$y' = \frac{(x^2+5)^2\left(6x(2x-1) - 2(x^2+5)\right)}{(2x-1)^2}$

$y' = \frac{(x^2+5)^2(12x^2-6x-2x^2-10)}{(2x-1)^2}$

$y' = \frac{(x^2+5)^2(10x^2-6x-10)}{(2x-1)^2}$

$\boxed{y' = \frac{2(5x^2-3x-5)(x^2+5)^2}{(2x-1)^2}}$

11. $y = \sin(2x)$

$y' = \cos 2x \cdot 2$

$\boxed{y' = 2\cos 2x}$

12. $y = \tan^2(3x) = [\tan(3x)]^2$

$y' = 2\tan(3x)\sec^2(3x)\cdot 3$

$\boxed{y' = 6\tan(3x)\sec^2(3x)}$

13. $y = \csc(2x)^3$

$y = \csc(8x^3)$

$y' = -\csc(8x^3)\cot(8x^3)(24x^2)$

$\boxed{y' = -24x^2\csc(8x^3)\cot(8x^3)}$

14. $y = \frac{x^2}{\cos x}$

$y' = \frac{\cos x(2x) - (x^2)(-\sin x)}{\cos^2 x}$

$\boxed{y' = \frac{2x\cos x + x^2\sin x}{\cos^2 x}}$

15. $y = x\cos x - \sin x$

$y' = x(-\sin x) + \cos x \cdot 1 - \cos x$

$y' = -x\sin x + \cos x - \cos x$

$\boxed{y' = -x\sin x}$

16. $y = \frac{\sin \pi x}{x+2}$ $y' = \frac{(x+2)\cos(\pi x)(\pi) - \sin(\pi x)}{(x+2)^2}$

$\boxed{y' = \frac{\pi(x+2)\cos \pi x - \sin \pi x}{(x+2)^2}}$

or $\boxed{y' = \frac{\pi x\cos \pi x + 2\pi\cos \pi x - \sin \pi x}{(x+2)^2}}$

17. $y = -2x^5 - 3x^3 + 2x - 10 \quad x = 2$

$y' = -10x^4 - 9x^2 + 2$

$y'(2) = -10(2)^4 - 9(2)^2 + 2$

$\boxed{y'(2) = -194}$

18. $y = \sin(2x) \quad (\pi/2, 0)$

$y' = \cos(2x)(2)$

$y' = 2\cos(2x)$

$y'(\pi/2) = 2\cos(2 \cdot \frac{\pi}{2})$

$y'(\pi/2) = 2\cos\pi = -2$

$\boxed{y - 0 = -2(x - \pi/2)}$

19. $f(x) = 2x^3 + 6x^2$

$f'(x) = -6$

$f'(x) = 8x^2 + 12x = -6$

$8x^2 + 12x - 6 = 0$

$2(4x^2 + 6x - 3) = 0$

* not factorable

$x = \frac{-6 \pm \sqrt{36 - 4(4)(-3)}}{2(4)}$

$x = \frac{-6 \pm \sqrt{84}}{8}$

$\boxed{x = 0.396, -1.896}$

20. $h(t) = -4.9t^2 + h$

A. when $h = 0, t = 9.2$ sec

$0 = -4.9(9.2)^2 + h$

$+414.736 = h$

$\boxed{+414.736 \text{ m}}$

B. $h(t) = -4.9t^2 + 414.736$

$h'(t) = -9.8t$

$h'(9.2) = -9.8(9.2)$

$\boxed{v(9.2) = -90.16 \text{ m/sec}}$

21. $x^2 + 3xy + y^3 = 10$

$2x + 3x\frac{dy}{dt} + y(3) + 3y^2\frac{dy}{dt} = 0$

$2x + 3x\frac{dy}{dt} + 3y + 3y^2\frac{dy}{dt} = 0$

$3x\frac{dy}{dt} + 3y^2\frac{dy}{dt} = -2x - 3y$

$dy/dt(3x + 3y^2) = -2x - 3y$

$\boxed{\frac{dy}{dt} = \frac{-2x - 3y}{3x + 3y^2}}$

22. $\cos(x+y) = x$

$-\sin(x+y)\left(1 + \frac{dy}{dx}\right) = 1$

$-\sin(x+y) - \sin(x+y)\frac{dy}{dx} = 1$

$-\sin(x+y)\,dy/dx = 1 + \sin(x+y)$

$\boxed{\frac{dy}{dx} = -\frac{1 + \sin(x+y)}{\sin(x+y)}}$

$\frac{dy}{dx} = \frac{-1}{\sin(x+y)} + \frac{\sin(x+y)}{\sin(x+y)}$

$\frac{dy}{dx} = -\csc(x+y) + 1$

$\boxed{\frac{dy}{dx} = 1 - \csc(x+y)}$

20. $h(t) = -4.9t^2 + 2$

A. when $h(t) = 0, t = 9.2$

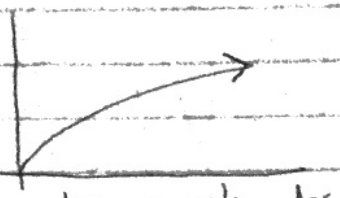
$0 = -9.8(9.2)^2 + h$

$829.472 = h$

B. $h(t) = -9.8t^2 + 829.472$

$h'(t) =$

23. $y = \sqrt{x}$



$\frac{dy}{dt} = 2 \qquad \frac{dy}{dt} = \frac{1}{2}x^{-\frac{1}{2}} \cdot \frac{dx}{dt}$

A. $x = 0.5$

$\frac{dy}{dt} = \frac{1}{2\sqrt{x}} \frac{dx}{dt}$

$2 = \frac{1}{2\sqrt{\frac{1}{2}}} \cdot \frac{dx}{dt}$

$\boxed{\frac{dy}{dt} = 12.828}$

B. $x = 1$

$2 = \frac{1}{2\sqrt{1}} \cdot \frac{dx}{dt}$

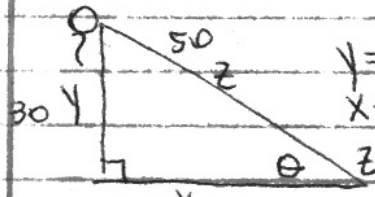
$\boxed{\frac{dy}{dt} = 4}$

C. $x = 4$

$2 = \frac{1}{2\sqrt{4}} \cdot \frac{dx}{dt}$

$\boxed{\frac{dy}{dt} = 8}$

24.




$y = 30 \quad dy/dt = 12 \quad \tan\theta = \frac{y}{x}$

$x = 40 \quad dx/dt = 0$

$z = 50 \quad dz/dt =$

$\theta = \quad d\theta/dt = ?$

$\tan\theta = \frac{y}{x} \qquad \sec^2\theta \frac{d\theta}{dt} = \frac{x(dy/dt) - y(dx/dt)}{x^2}$

$\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{(40)(12) - (30)(0)}{40^2}$

$\frac{25}{16} \frac{d\theta}{dt} = \frac{3}{10}$

$\boxed{\frac{d\theta}{dt} = 0.192 \text{ rad/sec}}$

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20. $h(t) = -16t^2 + 0t + h_0$

A. When $h = 0$, $t = 9.2$

$0 = -16(9.2)^2 + h_0$

$1352.24 \text{ ft} = h_0$

$\boxed{1352.24 \text{ ft}}$

B. $h'(t) = -32t$

$v(t) = -32t$

$v(9.2) = -32(9.2)$

$\boxed{v(9.2) = -294.4 \text{ ft/sec}}$