U5 Review

1. $f(x) = \ln\sqrt[5]{\frac{4x^2-1}{4x^2+1}} = \frac{1}{5}\left[\ln\left(\frac{4x^2-1}{4x^2+1}\right)\right] = \boxed{\frac{1}{5}\ln(4x^2-1) - \frac{1}{5}\ln(4x^2+1)}$

2. A. $f(x) = \sqrt[3]{x+1}$

$x = \sqrt[3]{y+1}$

$x^3 = y+1$

$\boxed{\begin{array}{l} y = x^3 - 1 \\ D: (-\infty, \infty) \end{array}}$

B. $f(x) = 3x^2 - 6$

$x = 3y^2 - 6$

$x + 6 = 3y^2$

$\frac{1}{3}(x+6) = y^2$

$y^2 = \frac{1}{3}(x+6)$

$\boxed{y = \sqrt{\tfrac{1}{3}x + 2} \quad D: [0, \infty)}$

3. A. $f(x) = x^3 - 9 \quad a = -1$

$f(g(x)) = x$

$f'(g(x)) \cdot g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(-1) = \frac{1}{f'(g(-1))}$

$g'(-1) = \frac{1}{f'(2)}$

$\boxed{g'(-1) = \frac{1}{12}}$

$x^3 - 9 = -1$

$x^3 = 8, \ x = 2$

$f'(x) = 3x^2$

$f'(2) = 3(2)^2 = 12$

B. $f(x) = x\sqrt{x-3} \quad a = 4$

$x\sqrt{x-3} = 4$ when $x = 4$

$g'(4) = \frac{1}{f'(g(4))} = \frac{1}{f'(4)} = \boxed{\frac{1}{3}}$

$f'(x) = x\left(\frac{1}{2}\right)(x-3)^{-1/2} + (x-3)^{1/2}$

$f'(x) = \frac{x}{2\sqrt{x-3}} + \sqrt{x-3}$

$f'(4) = \frac{4}{2\sqrt{4-3}} + \sqrt{4-3} = 2 + 1 = 3$

4. A. $f(x) = x\sqrt{\ln x} = x(\ln x)^{1/2}$

$f'(x) = x \cdot \frac{1}{2}(\ln x)^{-1/2}\left(\frac{1}{x}\right) + (\ln x)^{1/2}$

$f'(x) = \frac{1}{2\sqrt{\ln x}} + \sqrt{\ln x} \cdot \frac{2\sqrt{\ln x}}{2\sqrt{\ln x}}$

$f'(x) = \frac{1 + 2\ln x}{2\sqrt{\ln x}} = \boxed{\frac{1 + \ln x^2}{2\sqrt{\ln x}}}$

B. $f(x) = \ln\left[\frac{x(x+2)}{x+3}\right]$

$f(x) = \ln x + \ln(x+2) - \ln(x+3)$

$f'(x) = \frac{1}{x} + \frac{1}{x+2} - \frac{1}{x+3}$

$f'(x) = \frac{(x+2)(x+3)}{x(x+2)(x+3)} + \frac{x(x+3)}{x(x+2)(x+3)} - \frac{x(x+2)}{x(x+2)(x+3)}$

$f'(x) = \frac{x^2 + 5x + 6 + x^2 + 3x - x^2 - 2x}{x(x+2)(x+3)} = \boxed{\frac{x^2 + 6x + 6}{x(x+2)(x+3)}}$

C. $f(x) = x^3 e^{2x}$

$f'(x) = x^3 \cdot e^{2x}(2) + e^{2x}(3x^2)$

$f'(x) = 2x^3 e^{2x} + 3x^2 e^{2x}$

$\boxed{f'(x) = x^2 e^{2x}(2x+3)}$

D. $y = \ln(6-x^2)^8$

$y = 8\ln(6-x^2)$

$y' = \frac{8}{6-x^2} \cdot -2x = \boxed{\frac{-16x}{6-x^2}}$

E. $f(x) = \ln\left(\frac{e^x}{3+e^x}\right)$

$f(x) = \ln e^x - \ln(3+e^x)$

$f(x) = x - \ln(3+e^x)$

$f'(x) = 1 - \frac{1}{3+e^x}(e^x)$

$f'(x) = 1 - \frac{e^x}{3+e^x}$

$f'(x) = \frac{3+e^x}{3+e^x} - \frac{e^x}{3+e^x}$

$\boxed{f'(x) = \frac{3}{3+e^x}}$

F. $f(x) = x(4^{-x})$

$f'(x) = x(\ln 4 \cdot 4^{-x} \cdot -1) + 4^{-x}(1)$

$f'(x) = -x\ln 4 \cdot 4^{-x} + 4^{-x}$

$\boxed{f'(x) = -4^{-x}(x\ln 4 - 1)}$

G. $f(x) = \log_3 \sqrt{1-x} = \frac{1}{2}\log_3(1-x)$

$f'(x) = \frac{1}{2} \cdot \frac{(-1)}{\ln 3 \cdot (1-x)} = \boxed{\frac{-1}{2\ln 3(1-x)}}$

H. $y = \arcsin(7x)$

$\boxed{y' = \frac{7}{\sqrt{1-49x^2}}}$

I. $y = x\,\text{arccsc}\, x$

$y' = x \cdot \frac{-1}{|x|\sqrt{x^2-1}} + \text{arccsc}\, x$

$\boxed{y' = \frac{-x}{|x|\sqrt{x^2-1}} + \text{arccsc}\, x}$

J. $y = \frac{1}{2}\arctan(e^{2x})$

$y' = \frac{1}{2} \cdot \frac{e^{2x}(2)}{1+(e^{2x})^2}$

$\boxed{y' = \frac{e^{2x}}{1+e^{4x}}}$

5. $y^2 \ln x + 4y = 6$

$y^2\left(\frac{1}{x}\right) + \ln x(2y)\,dy/dx + 4\,dy/dx = 0$

$(2y\ln x + 4)\,dy/dx = \frac{-y^2}{x}$

$\frac{dy}{dx} = \frac{\frac{-y^2}{x}}{2y\ln x + 4} = \boxed{\frac{-y^2}{2xy\ln x + 4x}}$

6.

A. $\int \frac{1}{7x+2}\,dx$ $\quad u = 7x+2$

$\frac{1}{7}\int \frac{7}{7x+2}\,dx$ $\quad du = 7\,dx$

$\boxed{\frac{1}{7}\ln|7x+2| + C}$

B. $\int \frac{\ln\sqrt{x}}{x}\,dx = \int \frac{\frac{1}{2}\ln x}{x}\,dx = \frac{1}{2}\int \frac{\ln x}{x}\,dx$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$\frac{1}{2}\int u\,du = \frac{1}{2}\frac{u^2}{2} + C = \frac{1}{4}u^2 + C$

$\boxed{\frac{1}{4}(\ln x)^2 + C}$

C. $\int \frac{x^2+2x+5}{x^2+5}\,dx$

$\int \left[\frac{x^2+5}{x^2+5} + \frac{2x}{x^2+5}\right]dx$

$\int \left[1 + \frac{2x}{x^2+5}\right]dx$

$\boxed{x + \ln(x^2+5) + C}$

D. $\int_1^4 \frac{x+1}{x}\,dx = \int_1^4 \left(\frac{x}{x} + \frac{1}{x}\right)dx = \int_1^4 \left(1 + \frac{1}{x}\right)dx$

$x + \ln|x|\Big]_1^4 = (4 + \ln 4) - (1 + \ln 1) = \boxed{3 + \ln 4}$

E. $\int x e^{1-x^2}\,dx$ $\quad u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int -2x e^{1-x^2}\,dx$

$-\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C$

$\boxed{-\frac{1}{2}e^{1-x^2} + C}$

F. $\int \frac{e^{6x}}{e^{6x}+5}\,dx$ $\quad u = e^{6x}+5$

$du = 6e^{6x}\,dx$

$\frac{1}{6}\int \frac{6e^{6x}}{e^{6x}+5}\,dx = \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln|u| + C$

$\boxed{\frac{1}{6}\ln|e^{6x}+5| + C}$

G. $\int (x+1)\, 3^{(x+1)^2}\,dx$ $\quad u = (x+1)^2$

$du = 2(x+1)\,dx$

$\frac{1}{2}\int 2(x+1)\, 3^{(x+1)^2}\,dx$

$\frac{1}{2}\int 3^u\,du = \frac{1}{2}\left(\frac{1}{\ln 3}\right)\cdot 3^u + C$

$\boxed{\frac{1}{2\ln 3}\cdot 3^{(x+1)^2} + C}$

H. $\int \frac{x}{\sqrt{1-x^4}}\,dx = \int \frac{x}{\sqrt{1^2-(x^2)^2}}\,dx$ $\quad a = 1$ $\quad u = x^2$ $\quad du = 2x\,dx$

$\frac{1}{2}\int \frac{2x\,dx}{\sqrt{1-(x^2)^2}}$

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin\left(\frac{u}{1}\right) + C$

$\boxed{\frac{1}{2}\arcsin x^2 + C}$

I. $\int \frac{1}{16+x^2}\,dx = \int \frac{1}{4^2+x^2}\,dx$

$a = 4 \quad u = x \quad du = dx$

$\boxed{\frac{1}{4}\arctan\frac{x}{4} + C}$

J. $\int \frac{4-x}{\sqrt{4-x^2}}\,dx = \int \frac{4}{\sqrt{4-x^2}}\,dx - \int \frac{x}{\sqrt{4-x^2}}\,dx$

$4\arcsin\frac{x}{2} - \int \frac{x}{\sqrt{4-x^2}}\,dx$ $\quad u = 4-x^2 \quad du = -2x\,dx$

$4\arcsin\frac{x}{2} - \frac{-1}{2}\int \frac{-2x}{\sqrt{4-x^2}}\,dx$

$4\arcsin\frac{x}{2} + \frac{1}{2}\int u^{-1/2}\,du$

$4\arcsin\frac{x}{2} + \frac{1}{2}\frac{u^{1/2}}{1/2} + C = \boxed{4\arcsin\frac{x}{2} + \sqrt{4-x^2} + C}$

K. $\int \frac{x+5}{x^2+25}\,dx$

$\int \frac{x}{x^2+25}dx + 5\int \frac{1}{x^2+25}dx$

$u = x^2+25$ $\quad$ $u = x$ $\quad a = 5$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{2x}{x^2+25}dx + 5\int \frac{1}{x^2+5^2}dx$

$\frac{1}{2}\ln|x^2+25| + 5 \cdot \frac{1}{5}\arctan\frac{x}{5} + C$

$\boxed{\frac{1}{2}\ln|x^2+25| + \arctan\frac{x}{5} + C}$

or

$\boxed{\ln\sqrt{x^2+25} + \arctan\frac{x}{5} + C}$

L. $\int \frac{x}{16+x^2}\,dx$ $\quad$ $u = 16+x^2$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{2x\,dx}{16+x^2}$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|16+x^2| + C$

$\boxed{\ln\sqrt{16+x^2} + C}$

M. $\int \frac{\arctan\left(\frac{x}{2}\right)}{4+x^2}\,dx$ $\quad$ $u = \arctan\left(\frac{x}{2}\right)$

$du = \frac{\frac{1}{2}\,dx}{1+\left(\frac{x}{2}\right)^2} = \frac{dx}{2\left(1+\frac{x^2}{4}\right)} = \frac{dx}{2+\frac{x^2}{2}}$

$\int \frac{\arctan\left(\frac{x}{2}\right)dx}{4+x^2}$ $\quad$ $du = \frac{dx}{2+\frac{x^2}{2}} \cdot \frac{2}{2} = \frac{2dx}{4+x^2}$

$\frac{1}{2}\int \arctan\left(\frac{x}{2}\right) \cdot \frac{2dx}{4+x^2}$

$\frac{1}{2}\int u\,du = \frac{1}{2}\frac{u^2}{2} + C = \frac{1}{4}u^2 + C = \boxed{\frac{1}{4}\left(\arctan\left(\frac{x}{2}\right)\right)^2 + C}$