U3 Study Guide

1. A. $f(x) = x^2(2x^2-32)$

$f(x) = 2x^4 - 32x^2$

$f'(x) = 8x^3 - 64x$

$f'(x) = 8x(x^2-8)$

$\boxed{x=0}$ $x^2 = 8$

$\boxed{x = \pm\sqrt{8} \text{ or } \pm 2\sqrt{2}}$

B. $f(x) = \frac{4x}{x^2+1}$

$f'(x) = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)}$

$f'(x) = \frac{4x^2+4-8x^2}{x^2+1}$

$f'(x) = \frac{-4x^2+4}{x^2+1} = \frac{-4(x^2-1)}{x^2+1}$

$f'(x) = 0$ when $x^2 - 1 = 0$

$\boxed{x = \pm 1}$

$f'(x)$ always exists.

2. A. $f(x) = x^2 + 2x - 4$ $[-2, 1]$

$f'(x) = 2x + 2 = 0$

when $x = -1$

$f(-2) = -4$

$f(-1) = -5$

$f(1) = -1$

$\boxed{\text{absolute min } (-1,-5) \quad \text{absolute max } (1,-1)}$

B. $f(x) = x - 2 - \cos x$ $[0, 2\pi]$

$f'(x) = 1 + \sin x = 0$

when $\sin x = -1$

$x = \frac{3\pi}{2}$

$f(0) = -2 - (1) = -3$

$f(\frac{3\pi}{2}) = \frac{3\pi}{2} - 2 - 0 = \frac{3\pi}{2} - 2 \approx 2.712$

$f(2\pi) = 2\pi - 2 - 1 = 2\pi - 3 \approx 3.283$

$\boxed{\text{absolute max. } (2\pi, 2\pi-3) \quad \text{absolute min. } (0,-3)}$

3. a. abs min $(1,2)$

abs max $(4,5)$

b. no min, abs max $(4,5)$

c. abs min $(1,2)$, no max

d. no min or max

4. A. $f(x) = x^2 - 5x + 4$ $[1,4]$

f is continuous on $[1,4]$

and differentiable on $(1,4)$.

$f(1) = 0 = f(4)$ $\therefore$ Rolle's Thm applies

$f'(x) = 2x - 5 = 0$

when $\boxed{x = 5/2}$

B. $f(x) = \frac{x^2-1}{x}$ $[-1,1]$

$\boxed{f \text{ is not continuous at } x=0 \quad \therefore \text{ Rolle's Thm does not apply}}$

5. A. $f(x) = x(x^2 - x - 2)$ $[-1, 1]$

$f(x) = x^3 - x^2 - 2x$

$f'(x) = 3x^2 - 2x - 2$

$\frac{f(1) - f(-1)}{1 - -1} = \frac{-2}{2} = -1$

$f'(x) = 3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}, 1$ 1 is an endpt

$\therefore \boxed{x = -\frac{1}{3}}$

B. $f(x) = \frac{x+1}{x}$ $[\frac{1}{2}, 2]$

$f'(x) = \frac{x(1) - (x+1)}{x^2}$

$f'(x) = \frac{-1}{x^2}$

$\frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = \frac{-1.5}{1.5} = -1$

$f'(x) = \frac{-1}{x^2} = -1$

$x^2 = 1$

$x = \pm 1$

so $\boxed{x = 1}$

6. A. $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$

$f'(x) = 3x(x-4) = 0$

$x = 0, 4$ f': $+$ | $-$ | $+$; f: 0, 4

$\boxed{\text{f is increasing on } (-\infty, 0) \cup (4, \infty) \text{, decreasing on } (0, 4)}$

B. $f(x) = \frac{x+3}{x^2}$

$f'(x) = \frac{x^2(1) - (x+3)(2x)}{x^4}$

$f'(x) = \frac{x^2 - 2x^2 - 6x}{x^4} = \frac{-x^2 - 6x}{x^4}$

$f'(x) = \frac{-x(x+6)}{x^4} = \frac{-(x+6)}{x^3}$

$f'(x) = 0$ when $x = -6$

$f'(x)$ DNE when $x = 0$

$-$ | $+$ | $-$; -6, 0

$\boxed{\text{f is decreasing on } (-\infty, -6) \cup (0, \infty)}$

$\boxed{\text{f is increasing on } (-6, 0)}$

7. A. $f(x) = x^{2/3} - 4$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

$f'(x)$ DNE when $x = 0$

$-$ | $+$; 0

$\boxed{\text{f(x) has a rel. min at } (0, -4) \text{ b/c } f'(x) \text{ changes from neg. to pos @ } x = 0}$

B. $f(x) = x^4 - 6x^2$

$f'(x) = 4x^3 - 12x$

$f'(x) = 4x(x^2 - 3) = 0$

when $x = 0, \pm\sqrt{3}$

$-$ | $+$ | $-$ | $+$; $-\sqrt{3}$, 0, $\sqrt{3}$

$\boxed{\text{f(x) has a relative min @ } (-\sqrt{3}, -9) + (\sqrt{3}, -9) \text{ b/c } f'(x) \text{ changes from neg to pos at } x = \pm\sqrt{3}}$

$\boxed{\text{f(x) has a relative max at } (0, 0) \text{ b/c } f'(x) \text{ changes from pos. to neg at } x = 0}$

8. A. $f(x) = 2x^3 - 3x^2 - 12x + 5$

$f'(x) = 6x^2 - 6x - 12$

$f''(x) = 12x - 6$

$f''(x) = 0$ when $x = \frac{1}{2}$

f'' − | +

f CCD $\frac{1}{2}$ CCU

$f(x)$ is CCD on $(-\infty, \frac{1}{2})$
$f(x)$ is CCU on $(\frac{1}{2}, \infty)$
$f(x)$ has a POI at $(\frac{1}{2}, \frac{-3}{2})$

B. $f(x) = x(x+1)^{1/2}$ Domain $x \geq -1$

$f'(x) = x(\frac{1}{2})(x+1)^{-1/2} + (x+1)^{1/2}$

$f'(x) = \frac{x}{2(x+1)^{1/2}} + (x+1)^{1/2}$

$f'(x) = \frac{x}{2(x+1)^{1/2}} + (x+1)^{1/2} \cdot \frac{2(x+1)^{1/2}}{2(x+1)^{1/2}}$

$f'(x) = \frac{x + 2(x+1)}{2(x+1)^{1/2}} = \frac{3x+2}{2(x+1)^{1/2}}$

$f''(x) = \frac{2(x+1)^{1/2}(3) - (3x+2)(2)(\frac{1}{2})(x+1)^{-1/2}}{4(x+1)}$

$f''(x) = \frac{6(x+1)^{1/2} - \frac{3x+2}{(x+1)^{1/2}}}{4(x+1)}$

$f''(x) = \frac{\frac{(x+1)^{1/2}}{(x+1)^{1/2}} 6(x+1)^{1/2} - \frac{3x+2}{(x+1)^{1/2}}}{4(x+1)}$

$f''(x) = \frac{6(x+1) - (3x+2)}{4(x+1)^{3/2}} = \frac{6x+6-3x-2}{4(x+1)^{3/2}}$

$f''(x) = \frac{3x+4}{4(x+1)^{3/2}}$ $f''(x) = 0$ when $x = -4/3$

$f'(x)$ DNE when $x = -1$

DNE [−

$-4/3$ −1

$f(x)$ is CCD on $(-1, \infty)$
no pts of inflection

9. A. $f(x) = -(x-5)^2$

$f'(x) = -2(x-5) = -2x + 10$

$f'(x) = 0$ when $x = 5$

$f''(x) = -2$

$f''(5) < 0 \therefore f$ has a relative max. at $x = 5$

B. $f(x) = x^3 - 9x^2 + 27x$

$f'(x) = 3x^2 - 18x + 27$

$f'(x) = 0$ when $3x^2 - 18x + 27 = 0$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$

$f''(x) = 6x - 18$

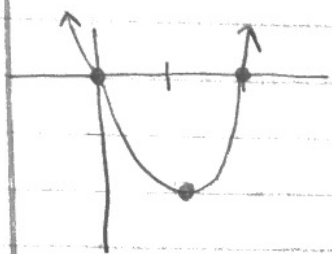
$f''(3) = 0$ and DT Fails

$f'(x) = 3(x-3)^2$ + | +

3

$f'(x)$ does not change sign at $x = 3 \therefore f$ has no rel. min. or max at $x = 3$

10.



11. A. $\lim_{x \to \infty} \frac{3-2x}{3x^3-1} = 0$

B. $\lim_{x \to \infty} \frac{3-2x}{3x-1} = \boxed{\frac{-2}{3}}$

C. $\lim_{x \to \infty} \frac{3-2x^2}{2x-1}$ DNE

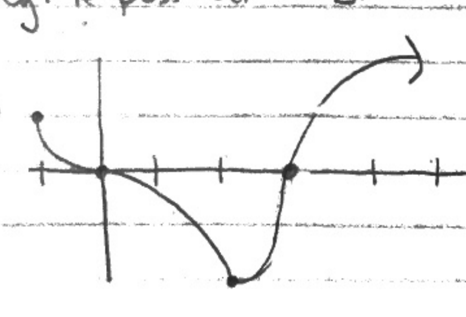
$$\begin{array}{r} -x - \frac{1}{2} \\ 2x-1 \,\overline{\big)\, -2x^2 + 0x + 3} \\ - \underline{\; -2x^2 + x \;} \\ -x + 3 \\ - \underline{\; -x + \frac{1}{2} \;} \\ 2.5 \end{array}$$

$\lim_{x \to \infty} -x - \frac{1}{2} + \frac{2.5}{2x-1} = \boxed{-\infty}$

*#12 on last page

13. a. f(x) has a relative minimum at $x = 2$ $(2,-2)$ because $f'(x)$ changes from neg. to pos. at $x=2$.

b.



14. $x = 1^{st}$ # $x = \frac{1}{y}$ $min = x+y$

$y = 2^{nd}$ #

$M = y^{-1} + y$

$M' = -y^{-2} + 1$

$M' = \frac{-1}{y^2} + 1 = \frac{-1+y^2}{y^2}$

$M' = \frac{y^2-1}{y^2} = 0$ when $y = \pm 1$

M' DNE when $y = 0$

$\boxed{\begin{array}{l} y=1 \\ x=1 \end{array}}$

15. $f(x) = x^2$ $(2, \frac{1}{2})$

$D = \sqrt{(x-2)^2 + (y-\frac{1}{2})^2}$

$D = \sqrt{(x-2)^2 + (x^2-\frac{1}{2})^2}$

$D = \sqrt{x^2 - 4x + 4 + x^4 - x^2 + \frac{1}{4}}$

$D = \sqrt{x^4 - 4x + 4.25}$

Let $g(x) = x^4 - 4x + 4.25$

$g'(x) = 4x^3 - 4 = 0$

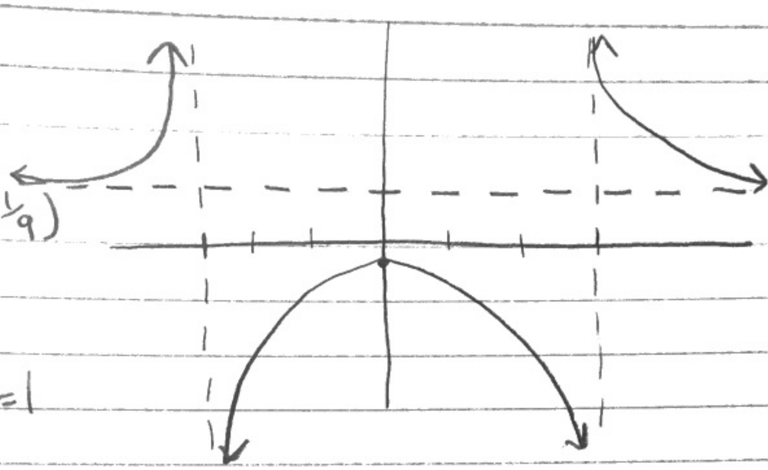
when $x = 1$

$g''(x) = 12x^2$

$g''(1) > 0$ ∴ min occurs at $x=1$ on D.

$f(1) = 1^2$ $\boxed{(1,1)}$

12. $f(x) = \frac{x^2+1}{x^2-9}$



A. Intercepts: $(0, -\frac{1}{9})$

B. Asymptotes:

VA @ $x = \pm 3$

HA $\lim_{x \to \infty} \frac{x^2+1}{x^2-9} = 1$

$y = 1$

C. $f'(x) = \frac{(x^2-9)(2x) - (x^2+1)(2x)}{(x^2-9)^2}$

$f'(x) = \frac{2x^3 - 18x - 2x^3 - 2x}{(x^2-9)^2}$

$f'(x) = \frac{-20x}{(x^2-9)^2}$

$f'(x) = 0$ when $x = 0$

$f'(x)$ DNE when $x = \pm 3$

f': + | − | + | −

f: −3 VA, 0, 3 VA

f is increasing on $(-\infty, -3) \cup (0, 3)$

decreasing on $(-3, 0) \cup (3, \infty)$

f has a rel. min at $(0, -\frac{1}{9})$

D. $f''(x) = \frac{(x^2-9)^2(-20) - (-20x)(2)(x^2-9)(2x)}{(x^2-9)^4}$

$f''(x) = \frac{-20(x^2-9) + 80x^2}{(x^2-9)^3} = \frac{-20x^2 + 180 + 80x^2}{(x^2-9)^3}$

$f''(x) = \frac{60x^2 + 180}{(x^2-9)^3}$

+ | − | +

−3, 3

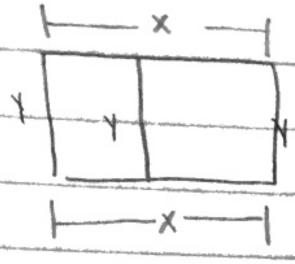
$f''(x) \neq 0$

$f''(x)$ DNE at $x = \pm 3$ f is ccu on $(-\infty, -3)(3, \infty)$

ccd on $(-3, 3)$

no pts of inflection

16. $P = 200 ft$



$A = xy$

$200 = 2x + 3y$

$x = \frac{200 - 3y}{2}$

$x = 100 - \frac{3}{2}y$

$A = (100 - \frac{3}{2}y)y$

$A = 100y - \frac{3}{2}y^2$

$A' = 100 - 3y = 0$

$y = 100/3 = 33\frac{1}{3}$

$y \approx 33.333$

$x = 100 - \frac{3}{2}(\frac{100}{3}) = 50$

$A'' = -3 < 0 \therefore A$ has a max at $y = 100/3$

$\boxed{\text{dimensions are } 50 \times \frac{100}{3}}$

17. $f(x) = x^5 \quad x = 2$

$f(2) = 32$

$f'(x) = 5x^4$

$f'(2) = 5(2)^4 = 80$

$y - 32 = 80(x - 2)$

$y - 32 = 80x - 160$

$y = 80x - 128$

$y(2.1) = 80(2.1) - 128 = \boxed{40}$