5B.3 DE's

p. 377 (31, 37, 41, 43-65 odds, omit 59)

31. $\frac{dy}{dx} = 3x^2$

$dy = 3x^2 dx$

$\int dy = \int 3x^2 dx$

$\boxed{y = x^3 + C}$

37. $\frac{dy}{dx} = \sin 2x$

$dy = \sin 2x\, dx$

$\int dy = \int \sin 2x\, dx \quad u = 2x$

$y = \frac{1}{2}\int 2\sin 2x\, dx \quad du = 2dx$

$y = \frac{1}{2}(-\cos 2x) + C$

$\boxed{y = -\frac{1}{2}\cos(2x) + C}$

41. $\frac{dy}{dx} = xe^{x^2}$

$dy = xe^{x^2} dx$

$\int dy = \int xe^{x^2} dx \quad u = x^2$

$y = \frac{1}{2}\int 2xe^{x^2} dx \quad du = 2x\, dx$

$\boxed{y = \frac{1}{2}e^{x^2} + C}$

43. $\frac{dy}{dx} = \frac{x}{y}$

$y\,dy = x\,dx$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + C$

$\boxed{y^2 - x^2 = C}$

45. $\frac{dr}{ds} = 0.05r$

$\frac{1}{r}dr = 0.05\,ds$

$\int \frac{1}{r}dr = \int 0.05\,ds$

$\ln|r| = 0.05s + C$

$e^{\ln|r|} = e^{0.05s + C}$

$\boxed{r = Ce^{0.05s}}$

47. $(2+x)\frac{dy}{dx} = 3y$

$\frac{1}{y}dy = \frac{3}{x+2}dx$

$\int \frac{1}{y}dy = 3\int \frac{1}{x+2}dx$

$\ln|y| = 3\ln|x+2| + C$

$e^{\ln|y|} = e^{\ln(x+2)^3 + C}$

$\boxed{y = C(x+2)^3}$

49. $y\frac{dy}{dx} = \sin x$

$y\,dy = \sin x\,dx$

$\int y\,dy = \int \sin x\,dx$

$\frac{1}{2}y^2 = -\cos x + C$

$\boxed{y^2 = -2\cos x + C}$

51. $\sqrt{1-4x^2}\,\frac{dy}{dx} = x$

$dy = \frac{x\,dx}{(1-4x^2)^{1/2}}$

$\int dy = \int x(1-4x^2)^{-1/2}dx \quad u = 1-4x^2 \quad du = -8x\,dx$

$y = -\frac{1}{8}\int 8x(1-4x^2)^{-1/2}dx$

$y = -\frac{1}{8}\frac{(1-4x^2)^{1/2}}{1/2} + C$

$y = -\frac{1}{4}(1-4x^2)^{1/2} + C$

$\boxed{y = -\frac{1}{4}\sqrt{1-4x^2} + C}$

53. $y \ln x - x\frac{dy}{dx} = 0$

$-x\frac{dy}{dx} = -y\ln x$

$x\,dy = y\ln x\,dx$

$\frac{1}{y}dy = \frac{\ln x}{x}dx$

$\int \frac{1}{y}dy = \int \frac{\ln x}{x}dx \quad u = \ln x,\ du = \frac{1}{x}dx$

$\ln|y| = \frac{(\ln x)^2}{2} + C$

$\boxed{y = Ce^{\frac{1}{2}(\ln x)^2}}$

55. $yy' - e^x = 0 \quad y(0) = 4$

$y\frac{dy}{dx} = e^x$

$y\,dy = e^x dx$

$\int y\,dy = \int e^x dx$

$\frac{1}{2}y^2 = e^x + C$

$y^2 = 2e^x + C$

$4^2 = 2e^0 + C$

$16 = 2 + C$

$14 = C$

$\boxed{y^2 = 2e^x + 14}$

57. $y(x+1) + \frac{dy}{dx} = 0 \quad y(-2) = 1$

$\frac{dy}{dx} = -y(x+1)$

$\frac{1}{y}dy = (-x-1)dx$

$\int \frac{1}{y}dy = \int (-x-1)dx$

$\ln|y| = -\frac{1}{2}x^2 - x + C$

$y = e^{-\frac{1}{2}x^2 - x + C}$

$y = Ce^{-\frac{1}{2}x^2 - x}$

$\boxed{y = e^{-\frac{x^2}{2} - x}}$

$1 = Ce^{-\frac{1}{2}(-2)^2 - (-2)}$

$1 = Ce^{-2+2}$

$1 = C$

61. $\frac{du}{dv} = uv\sin v^2$

$\frac{1}{u}du = v\sin v^2 dv$

$\int \frac{1}{u}du = \int v\sin v^2 dv$

$\ln|u| = \frac{1}{2}\int 2v\sin(v^2)dv$

$\ln|u| = \frac{1}{2}(-\cos v^2) + C$

$u = e^{-\frac{1}{2}\cos v^2 + C}$

$u = Ce^{-\frac{1}{2}\cos v^2} \quad u(0) = 1$

$1 = Ce^{-\frac{1}{2}\cos 0^2}$

$1 = Ce^{-\frac{1}{2}}$

$e^{\frac{1}{2}} = C$

$u = e^{\frac{1}{2}} \cdot e^{-\frac{1}{2}\cos v^2}$

$u = e^{\frac{1}{2} - \frac{1}{2}\cos v^2}$

$\boxed{u = e^{\frac{1}{2}(1 - \cos v^2)}}$

63. $dP - kP\,dt = 0$

$dP = kP\,dt$

$\frac{1}{P}dP = k\,dt$

$\int \frac{1}{P}dP = \int k\,dt$

$\ln|P| = kt + C$

$P = e^{kt+C}$

$P = Ce^{kt} \quad P(0) = P_0$

$P_0 = Ce^{k(0)}$

$P_0 = C$

$\boxed{P = P_0 e^{kt}}$