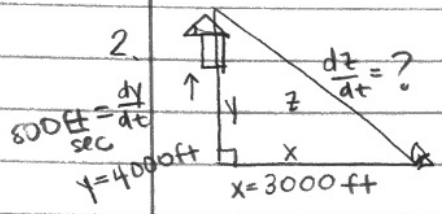
Related Rates

1. $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\quad \boxed{\frac{dr}{dt} = 0.796 \text{ cm/sec}}$

$dV/dt = 10 \text{ cm}^3/\text{sec}$ $\quad 10 = 4\pi (1)^2 \frac{dr}{dt}$

$dr/dt = ?$ when $r = 1$ cm $\quad \frac{10}{4\pi} = \frac{dr}{dt}$

2. 

$\frac{dy}{dt} = 800 \frac{ft}{sec}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$

$x^2 + y^2 = z^2$

$3000^2 + 4000^2 = z^2$

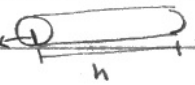
$z = 5000$

~~$(3000)(0)$~~ $+ (4000)(800) = (5000)\frac{dz}{dt}$

$\frac{(4000)(800)}{5000} = \frac{dz}{dt}$ $\quad \boxed{\frac{dz}{dt} = 640 \frac{ft}{sec}}$

3. $V = \pi r^2 h$ $\quad \frac{dr}{dt} = -4 \text{ ft/min}$ $\quad dh/dt = 2 \text{ ft/min}$

Find dV/dt when $r = 2$, $h = 6$



$dV/dt = \pi r^2 \frac{dh}{dt} + h(2\pi r)\frac{dr}{dt}$

$dV/dt = \pi (2)^2 (2) + 6(2\pi \cdot 2)(-4)$

$dV/dt = 8\pi - 96\pi$

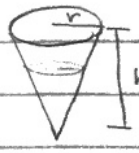
$dV/dt = -88\pi \text{ ft/min}$

$\boxed{dV/dt = 276.460 \text{ ft/min}}$

5. $V = \frac{1}{3}\pi r^2 h$ $\quad h = 120$ $\quad r = 40$

$dV/dt = 60 \text{ ft}^3/\text{min}$

? dh/dt when $h = 10$ ft



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{9} \cdot h$

$V = \frac{1}{27}\pi h^3$

$\frac{h}{r} = \frac{120}{40} = \frac{3}{1}$, $\quad h = 3r$, $\quad r = \frac{1}{3}h$

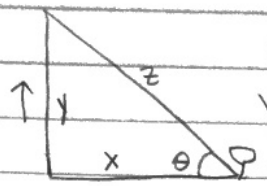
$dV/dt = \frac{1}{27} \cdot 3\pi h^2 \, dh/dt$

$60 = \left(\frac{1}{9}\right)\pi (10)^2 \, dh/dt$

$60 = \frac{100\pi}{9} \, dh/dt$

$dh/dt = 60 \cdot \frac{9}{100\pi} = \boxed{1.719 \text{ ft/min}}$

6.



$x = 10$ $dx/dt = 0$

$y = \sqrt{525}$ $dy/dt = 2^{m}/sec$

$z = 25$

$d\theta/dt = ?$

$\tan\theta = \frac{y}{x}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x\,dy/dt - y\,dx/dt}{x^2}$

$\left(\frac{5}{2}\right)^2 \frac{d\theta}{dt} = \frac{(10)(2) - (\sqrt{525})(0)}{100}$

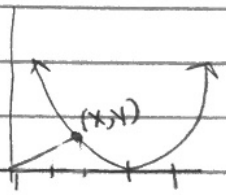
$\sec\theta = \frac{25}{10} = \frac{5}{2}$

$\frac{25}{4}\frac{d\theta}{dt} = \frac{1}{5}$

$d\theta/dt = \frac{1}{5} \cdot \frac{4}{25} = 0.032$

$\boxed{d\theta/dt = 0.032^{rad/sec}}$

7.



$dx/dt = 4^{u}/sec$

$x = 1$

A. ? dy/dt

If $x=1$, $y=4$

$y = (x-3)^2$

$dy/dt = 2(x-3)\,dx/dt$

$dy/dt = 2(1-3)(4)$

$\boxed{dy/dt = -16^{units/sec}}$

B. Distance from origin

$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$D = \sqrt{(x-0)^2 + (y-0)^2}$

$D = \sqrt{x^2+y^2}$

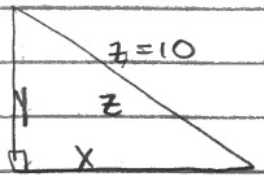
$dD/dt = \frac{1}{2}(x^2+y^2)^{-1/2}(2x\,dx/dt + 2y\,dy/dt)$

$dD/dt = \frac{1}{2}(1^2+4^2)^{-1/2}(2\cdot1\cdot4 + 2\cdot4\cdot-16)$

$dD/dt = \frac{1}{2}(17)^{-1/2}(8 + -128)$

$\boxed{dD/dt \approx -14.552^{unit/sec}}$

8.



$x = 6$ $dx/dt = 1^{ft/sec}$

$y = 8$ $dy/dt = ?$

$z = 10$ $dz/dt = 0$

$x^2+y^2 = z^2$

$2x\,dx/dt + 2y\,dy/dt = 2z\,dz/dt$

$x\,dx/dt + y\,dy/dt = z\,dz/dt$

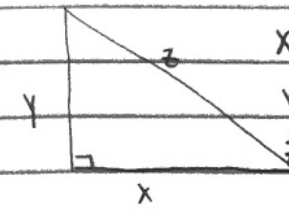
$(6)(1) + 8\,dy/dt = (10)(0)$

$6 + 8\,dy/dt = 0$

$dy/dt = -6/8$

$\boxed{dy/dt = -0.75^{ft/sec}}$

9.



$x = 2.5$ $dx/dt = 0$

$y = 1$ $dy/dt = 750$

$z = \sqrt{7.25}$ $dz/dt = ?$

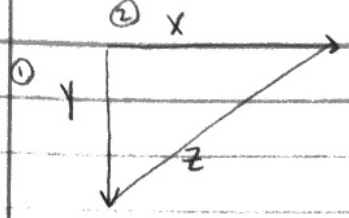
$x^2+y^2 = z^2$

$2x\,dx/dt + 2y\,dy/dt = 2z\,dz/dt$

$x\,dx/dt + y\,dy/dt = z\,dz/dt$

$(2.5)(0) + (1)(750) = \sqrt{7.25}\,dz/dt$

$\frac{dz}{dt} = \frac{750}{\sqrt{7.25}} = \boxed{278.543^{miles/hour}}$

11. 

$X = 900$ $\quad dx/dt = 400$ $\quad x^2 + y^2 = z^2$

$Y = 900$ $\quad dy/dt = -450$ $\quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$z = \sqrt{1620000}$ $\quad dz/dt = ?$

Plane 1: 450 m/h for 2 hours = 900 miles

Plane 2: 600 m/h for 1.5 hours = 900 miles

$x\,dx/dt + y\,dy/dt = z\,dz/dt$

$(900)(600) + (900)(-450) = (\sqrt{1620000})\frac{dz}{dt}$

$\frac{135000}{\sqrt{1620000}} = \frac{dz}{dt} = \boxed{106.0660 \text{ mph}}$

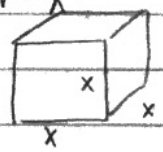
12. $V = \frac{4}{3}\pi r^3$ $\quad dV/dt = 4\pi r^2 dr/dt$

$dV/dt = 5 \text{ cm}^3/\text{s}$ $\quad 5 = 4\pi(15)^2 dr/dt$

? dr/dt when $r = 15$ $\quad \frac{dr}{dt} = \frac{5}{4\pi(15)^2} = \boxed{0.002 \text{ cm/sec}}$

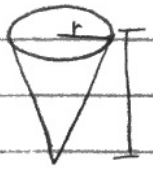
13. Same as #5

14. $V = x^3$ $\quad dx/dt = 2$ $\quad dV/dt = 3x^2\,dx/dt$



$x = 3$ $\quad dV/dt = 3(3)^2(2)$

? dV/dt $\quad \boxed{dV/dt = 54 \text{ in}^3/\text{min}}$

15. 

$r = 2$ $\quad dV/dt = 2 \text{ m}^3/\text{min}$ $\quad V = \frac{1}{3}\pi r^2 h$

$h = 4$ $\quad dh/dt = ?$ $\quad V = \frac{1}{3}\pi(\frac{1}{2}h)^2 h$

$h = 3$ $\quad V = \frac{1}{12}\pi h^3$

$\frac{r}{h} = \frac{2}{4}$ $\quad 4r = 2h$ $\quad dV/dt = \frac{1}{4}\pi h^2\,dh/dt$

$r = \frac{1}{2}h$ $\quad 2 = \frac{1}{4}\pi(3)^2\,dh/dt$

$2 = \frac{9\pi}{4}\,dh/dt$

$2 \cdot \frac{4}{9\pi} = dh/dt = \boxed{0.283 \text{ m/min}}$

16.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{3}{4}h\right)^2 h$

$V = \frac{3}{16}\pi h^3$

$h = 4 \quad r = 3$

$\frac{h}{r} = \frac{4}{3}$

$3h = 4r$

$r = \frac{3}{4}h$

$dV/dt = 2 \text{ in}^3/\text{min}$

$dh/dt = ?$ $\quad h = 26 \text{ in}$

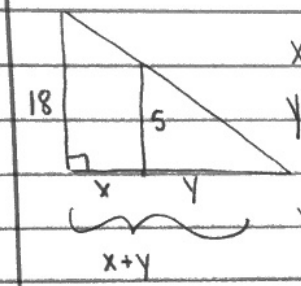
$dV/dt = \frac{9}{16}\pi h^2 \frac{dh}{dt}$

$2 = \frac{9}{16}\pi (26)^2 \, dh/dt$

$0.002 = dh/dt$

$\boxed{0.002 \text{ in/min} = dh/dt}$

17.



$x = 30 \quad dx/dt = -6$

$y = \frac{150}{13} \quad dy/dt = ?$

$\frac{y}{5} = \frac{x+y}{18}$

$18y = 5x + 5y$

$13y = 5x$

$y = \frac{5}{13}x$

$y = \frac{5}{13}(30)$

$dy/dt = 5/13 \, dx/dt$

$= (5/13)(-6) = -30/13$

$\boxed{dy/dt = -2.308 \text{ ft/sec}}$

18. $V = \frac{1}{12\pi} C^2 h$

A. $dV/dt = \frac{1}{12\pi} C^2 \, dh/dt + h \cdot \frac{1}{6\pi} C \, dC/dt$

The rate at which the volume of the tree changes over time.

B. $dC/dt = 0.02 \text{ m/y}$

$dh/dt = 4$

$h = 22$

$C = 5$

$dV/dt = \frac{1}{12\pi}(5)^2(4) + (22)\left(\frac{1}{6\pi}\right)(5)(0.02)$

$dV/dt = 25/3\pi + 11/30\pi$

$\boxed{dV/dt = 2.7609 \text{ m}^3/\text{year}}$

19. $r = 10 \text{ cm}$

$V = \frac{4}{3}\pi r^3$

$dV/dt = 4\pi r^2 \, dr/dt$

$dV/dt = 400\pi \, dr/dt$

The volume is changing 400π times faster.

20. $dr/dt = 0.5$

$? \; dV/dt$

$r = 6$

$dV/dt = 4\pi r^2 \, dr/dt$

$dV/dt = 4\pi (6)^2 (0.5)$

$\boxed{dV/dt = 226.195 \text{ cm}^3/\text{sec}}$