Logstic. HW Answer Key

1. $\int \frac{3}{(x-1)(x+2)}\,dx \qquad \frac{3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$3 = A(x+2) + B(x-1)$

$\int \left(\frac{1}{x-1} + \frac{-1}{x+2}\right)dx$ If $x=1$, $3 = A(3) + B(0) \therefore A = 1$

$x=-2$, $3 = A(0) + B(-3) \therefore B = -1$

$\ln|x-1| - \ln|x+2| + C$

$\boxed{\ln\left|\frac{x-1}{x+2}\right| + C}$

2. $\int \frac{1}{x^2-4}\,dx = \int \frac{dx}{(x+2)(x-2)} \qquad \frac{1}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$

$1 = A(x-2) + B(x+2)$

$\int \left(\frac{-1}{4}\cdot\frac{1}{x+2} + \frac{1}{4}\cdot\frac{1}{x-2}\right)dx$ If $x=2$, $1 = A(0) + B(4) \therefore B = \frac{1}{4}$

$x=-2$, $1 = A(-4) + B(0) \therefore A = -\frac{1}{4}$

$-\frac{1}{4}\ln|x+2| + \frac{1}{4}\ln|x-2| + C$

$\frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$

$\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$

$\boxed{\ln\sqrt[4]{\frac{x-2}{x+2}} + C}$

3. $\int \frac{x^2-1}{x^2-16}\,dx = \int \frac{x^2-1}{(x+4)(x-4)}\,dx \qquad \frac{x^2-1}{(x+4)(x-4)} = \frac{A}{x+4} + \frac{B}{x-4}$

$x^2-1 = A(x-4) + B(x+4)$

$\int \left(\frac{-15}{8}\cdot\frac{1}{x+4} + \frac{15}{8}\cdot\frac{1}{x-4}\right)dx$ If $x=4$, $15 = A(0) + B(8) \therefore B = \frac{15}{8}$

$x=-4$, $15 = A(-8) + B(0) \therefore A = \frac{-15}{8}$

$-\frac{15}{8}\ln|x+4| + \frac{15}{8}\ln|x-4| + C$

$\boxed{\frac{15}{8}\ln\left|\frac{x-4}{x+4}\right| + C}$

4. $\int_1^2 \frac{3}{x^2+x}\,dx = \int_1^2 \frac{3}{x(x+1)}\,dx$ $\qquad \frac{3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$

$\int_1^2 \left(\frac{3}{x} - \frac{3}{x+1}\right)dx$ $\qquad 3 = A(x+1) + Bx$

If $x=0$, $3 = A(1) + B(0)$ $\therefore A = 3$

$x=-1$, $3 = A(0) + B(-1)$ $\therefore B = -3$

$3\ln|x| - 3\ln|x+1| \Big]_1^2$

$3\ln\left|\frac{x}{x+1}\right| \Big]_1^2 = \ln\left(\frac{x}{x+1}\right)^3 \Big]_1^2 = \ln\left(\frac{2}{2+1}\right)^3 - \ln\left(\frac{1}{1+1}\right)^3$

$\ln\left(\frac{8}{27}\right) - \ln\left(\frac{1}{8}\right) = \boxed{\ln\left(\frac{64}{27}\right) \approx 0.863}$

5. $y = \frac{600}{1+59e^{-0.1t}}$ $\boxed{300}$ the rate is greatest at half the carrying capacity

6. $y = \frac{0.9}{1+45e^{-0.15t}}$ If $L = 0.9$ or 90%, $\boxed{10\% \text{ will not be affected}}$

7. $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$ $\qquad \frac{dP}{dt} = 2P\left(1 - \frac{P}{10000}\right)$ $\qquad \boxed{\lim_{t\to\infty} P(t) = 10{,}000}$

8. $P(t) = \frac{2100}{1+e^{4.3-t}} = \frac{2100}{1+e^{4.3}\cdot e^{-t}}$ $\qquad$ So $C = e^{4.3}$

A. $k = 1$, Carrying Capacity $= 2100$

B. $P(0) = \frac{2100}{1+e^{4.3-0}} \approx 28.112$ $\boxed{28 \text{ students}}$

C. $400 = \frac{2100}{1+e^{4.3-t}}$ $\qquad t \approx 2853$ days

9. $\frac{dP}{dt} = 3P - 0.01P^2$ $\frac{dP}{dt} = 3P\left(1 - \frac{P}{300}\right)$

A + B only **C is false** If $P > 300$, $\frac{dP}{dt}$ is decreasing

10. $\frac{dP}{dt} = 0.01P(100 - P)$ $\frac{dP}{dt} = P\left(1 - \frac{P}{100}\right)$

A. $P(t) = \frac{100}{1 + Ce^{-t}}$ If $P(0) = 20$, $20 = \frac{100}{1 + Ce^{0}}$ $1 + C = 5$ $C = 4$

$P(t) = \frac{100}{1 + 4e^{-t}}$

B. $P(3) = \frac{100}{1 + 4e^{-3}}$ ≈ 83.392 animals

C. $80 = \frac{100}{1 + 4e^{-t}}$

$1 + 4e^{-t} = \frac{100}{80}$

$1 + 4e^{-t} = \frac{5}{4}$

$4e^{-t} = \frac{5}{4} - 1$

$4e^{-t} = \frac{1}{4}$

$e^{-t} = \frac{1}{16}$

$-t = \ln\left(\frac{1}{16}\right)$

$t = -\ln\left(\frac{1}{16}\right)$

$t = \ln 16$ ≈ 2.772 years