Unit 1 AP Calculus AB
Limits Review Guide

Name Key

**Evaluating Limits:**

1. Numerically – create a table of values
2. Graphically – observe a graph
3. Algebraically (Analytically)

*2 Trig. Limits $\lim_{x\to 0} \frac{\sin x}{x} = 1$ $\quad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0$

**Strategies for Evaluating Limits Analytically:**

1. direct substitution
2. factor and cancel
3. multiply by conjugate
4. common denominator

Problems: Find the limit (if it exits).

1. $\lim_{y\to 4} 3|y-1| = 3|4-1|$

$3|3| = \boxed{9}$

2. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$\lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)}$

$\lim_{x\to 4} \frac{1}{\sqrt{x}+2} =$

$\frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

3. $\lim_{x\to -2} \frac{x^2-4}{x^3+8}$

$\lim_{x\to -2} \frac{(x+2)(x-2)}{(x+2)(x^2-2x+4)}$

$\lim_{x\to -2} \frac{x-2}{x^2-2x+4} =$

$\frac{-2-2}{(-2)^2-2(-2)+4} = \frac{-4}{12}$

$\boxed{-\frac{1}{3}}$

4. $\lim_{s\to 0} \frac{(1/\sqrt{1+s})-1}{s} = \lim_{s\to 0} \frac{\frac{1}{\sqrt{1+s}} - 1}{s}$

$\lim_{s\to 0} \frac{\frac{1}{\sqrt{1+s}} - \frac{\sqrt{1+s}}{\sqrt{1+s}}}{s}$

$\lim_{s\to 0} \frac{\frac{1-\sqrt{1+s}}{\sqrt{1+s}}}{s}$

$\lim_{s\to 0} \frac{1-\sqrt{1+s}}{\sqrt{1+s}} \cdot \frac{1}{s}$

$\lim_{s\to 0} \frac{1-\sqrt{1+s}}{s\sqrt{1+s}} \cdot \frac{1+\sqrt{1+s}}{1+\sqrt{1+s}}$

$\lim_{s\to 0} \frac{1-(1+s)}{s\sqrt{1+s}(1+\sqrt{1+s})}$

$\lim_{s\to 0} \frac{-s}{s\sqrt{1+s}(1+\sqrt{1+s})}$

$\lim_{s\to 0} \frac{-1}{\sqrt{1+s}(1+\sqrt{1+s})} = \frac{-1}{\sqrt{1}(1+\sqrt{1})} = \boxed{-\frac{1}{2}}$

5. $\lim_{x\to 0} \frac{1-\cos x}{\sin x} = \lim_{x\to 0} \frac{1-\cos x}{1} \cdot \frac{1}{\sin x} =$

$\lim_{x\to 0} \frac{1-\cos x}{x} \cdot \frac{x}{\sin x}$

multiplied by $\frac{x}{x}$

$\lim_{x\to 0} \frac{1-\cos x}{x} \cdot \frac{x}{\sin x}$

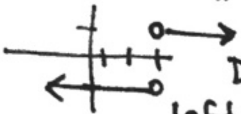
$0 \cdot 1 = \boxed{0}$

Functions for which you must check the limit numerically from the left and right:

1. Absolute value
2. Piecewise
3. Rational

Problems: Find the limit (if it exists). If the limit does not exist, explain why.

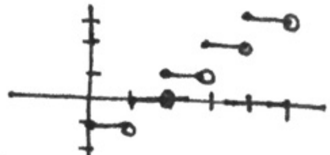
6. $\lim\limits_{x \to 3} \dfrac{|x-3|}{x-3}$



Does not exist b/c left and right hand limits are different.

$\lim\limits_{x \to 3^+} f(x) = 1 \quad \lim\limits_{x \to 3^-} f(x) = -1$

7. $\lim\limits_{x \to 4} [\![x-1]\!]$



Does not exist because left and right hand limits are different.

$\lim\limits_{x \to 4^-} f(x) = 2 \quad \lim\limits_{x \to 4^+} f(x) = 3$

8. $\lim\limits_{x \to 2} f(x)$, where $f(x) = \begin{cases} (x-2)^2, & x \le 2 \\ 2-x, & x > 2 \end{cases}$

$\lim\limits_{x \to 2^-} (x-2)^2 = 0$

$\lim\limits_{x \to 2^+} 2-x = 0$

Therefore,

$\lim\limits_{x \to 2} f(x) = 0$

9. $\lim\limits_{t \to 1} h(t)$, where $h(t) = \begin{cases} t^3+1, & t<1 \\ \frac{1}{2}(t+1), & t \ge 1 \end{cases}$

$\lim\limits_{t \to 1^-} t^3+1 = 2$

$\lim\limits_{t \to 1^+} \frac{1}{2}(t+1) = 1$

$\therefore \lim\limits_{t \to 1} h(t)$ DNE

(different left and right hand limits)

**Continuity** – There are 3 qualifications that functions must satisfy in order for them to be continuous.

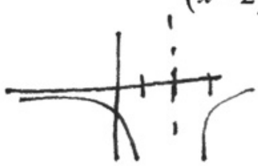
1. $f(c)$ must exist
2. $\lim\limits_{x \to c} f(x)$ must exist
3. $\lim\limits_{x \to c} f(x) = f(c)$

There are 2 types of discontinuities:

1. removable – holes
2. nonremovable – jumps, gaps, asymptotes

Determine the intervals on which the function is continuous. Identify the type of any discontinuities.

10. $f(x) = \dfrac{1}{(x-2)^2}$



NR @ $x = 2$

11. $f(x) = \dfrac{3x^2 - x - 2}{x - 1}$

$f(x) = \dfrac{(3x+2)(x-1)}{(x-1)}$

R @ $x = 1$

12. $f(x) = \begin{cases} 5 - x, & x \le 2 \\ 2x - 3, & x > 2 \end{cases}$

$\lim\limits_{x \to 2^-} 5 - x = 3$

$\lim\limits_{x \to 2^+} 2x - 3 = 1$

NR @ $x = 2$

13. Determine the value of c such that the function $f(x) = \begin{cases} x + 3, & x \le 2 \\ cx + 6, & x > 2 \end{cases}$ is continuous over all reals.

$x + 3 = cx + 6$ when $x = 2$

$2 + 3 = c(2) + 6$

$5 = 2c + 6$ $\quad \boxed{c = -\frac{1}{2}}$

$-1 = 2c$

**Intermediate Value Theorem** – Hypothesis: $f(x)$ must be continuous on $[a,b]$; $f(a) < k < f(b)$

Conclusion: there must be a c in $[a,b]$ such that $f(c) = k$

14. Use the Intermediate Value Theorem to show that for $f(x) = x^2 + x - 1$ on $[0,5]$ there exists a c such that $f(c) = 11$. Then find the value(s) of c guaranteed by the theorem.

$f(x)$ is continuous on $[0,5]$

$f(0) = -1$ $\quad f(0) < 11 < f(5)$

$f(5) = 19$ therefore a value of c must exist such that $f(c) = 11$

$11 = x^2 + x - 1$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x = -4, 3$

$\boxed{c = 3}$

**Vertical Asymptotes:** A function with $\lim_{x \to c} f(x) = \underline{\pm\infty}$ has a vertical asymptote at $\underline{x = c}$.

Rational Functions – Check where denominator = $\underline{0}$

Numerator cannot = $\underline{0}$.

Vertical Asymptotes occur at NON REMOVABLE discontinuities.

15. Find the vertical asymptote(s) (if any exist) of the function. $h(x) = \dfrac{x^2 - 2x}{x^4 - 16} = \dfrac{x(x-2)}{(x^2-4)(x^2+4)}$

$\boxed{\begin{array}{l} R\ @\ x = 2 \\ NR\ @\ x = -2 \end{array}}$

$h(x) = \dfrac{x(x-2)}{(x^2+4)(x+2)(x-2)}$

**One-sided Infinite Limits** – The limit will equal $\frac{\#}{0}$. Use direct substitution and then check numerically to get the sign of infinity.

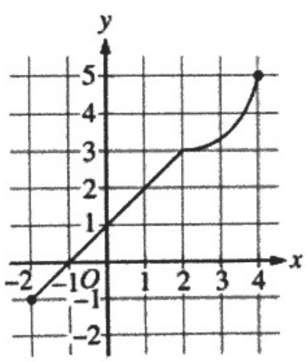
16. $\lim_{x \to 1^-} \dfrac{x^2 + 2x + 1}{x - 1} = \lim_{x \to 1^-} \dfrac{\overset{+}{(x+1)}\overset{+}{(x+1)}}{(x - 1)} = -\infty$

17. $\lim_{x \to 0^+} \dfrac{\csc 2x}{x} = \lim_{x \to 0^+} \dfrac{1}{x \sin(2x)} = \boxed{\infty}$

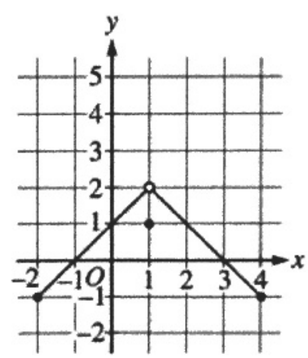
* just right of 0, sin (2x) is positive and x is positive.

**Sample AP Question –**

**A calculator may not be used on questions on this part of the exam.**



Graph of $f$



Graph of $g$

C 1. The graphs of the functions $f$ and $g$ are shown above. The value of $\lim_{x \to 1} f(g(x))$ is

(A) 1

(B) 2

(C) 3

(D) nonexistent

$\lim_{x \to 1} g(x) = 2$

$\lim_{x \to 2} f(x) = 3$