#1 Limits and Continuity Review

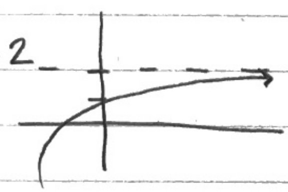
B 1. $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)} = -2$ If degree of numerator = degree of denominator, the limit is the ratio of the leading coefficients.

A 3 $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 1, & x = 2 \end{cases}$ $\lim_{x\to 2} \frac{x^2-4}{x-2} \overset{L'H}{=} \lim_{x\to 2} \frac{2x}{1} = 4$

but $f(2) = 1$ so f is not continuous.

A 2. $\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2} = \lim_{x\to 0} \frac{\cancel{x^2}(5x^2+8)}{\cancel{x^2}(3x^2-16)} = \frac{5(0)^2+8}{3(0)^2-16} = \boxed{\frac{-1}{2}}$

E 4. HA @ $y=2$



$\lim_{x\to\infty} f(x) = 2$

C 5. $\lim_{x\to\infty} \frac{x^3-2x^2+3x-2}{4x^3-3x^2+2x-1} = \frac{1}{4}$ see #1

A 6. sharp points are continuous but not differentiable

D 7. $f(x) = \begin{cases} x+2, & x \leq 3 \\ 4x-7, & x > 3 \end{cases}$ $\lim_{x\to 3^-} f(x) = 3+2 = 5$ $f(3) = 3+2 = 5$

$\lim_{x\to 3^+} f(x) = 4(3) - 7 = 5$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$ $\therefore$ f is continuous at $x=3$ and $\lim_{x\to 3} f(x)$ exists

* $f(x)$ is not differentiable because there is a sharp point at $x=3$

C 8. the left and right hand limits exist but are not equal.

E 9. $f(-2) = f(2) = 0$

$f'(c) \neq 0$

D 10.

B 11. $\lim_{x \to \pi} \frac{\cos x + \sin(2x) + 1}{x^2 - \pi^2} \overset{LH}{=} \lim_{x \to \pi} \frac{-\sin x + 2\cos(2x)}{2x} =$ L'Hopital's Rule

$= \frac{-\sin(\pi) + 2\cos(2\pi)}{2(\pi)} = \frac{-(0) + 2(1)}{2\pi} = \frac{2}{2\pi} = \boxed{\frac{1}{\pi}}$

C 12. $\lim_{x \to 0} \frac{(1+x)^6 - 1}{x} = \lim_{x \to 0} \frac{6(1+x)^5}{1} = 6(1+0)^5 = 6$

B 13. $\lim_{x \to 0} \frac{\cos x - 1}{x} = \lim_{x \to 0} \frac{-\sin x}{1} = \frac{0}{1} = \boxed{0}$