HW #1 – Limits and Continuity Review

1. (NC - 2008) $\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

   A. -3 B. -2 C. 2 D. 3 E. Nonexistent

2. (NC – 2008) $\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$ is

   A. -1/2 B. 0 C. 1 D. 5/3 + 1 E. Nonexistent

3. (NC – 2008) Let ***f*** be the function defined below. Which of the following statements about ***f*** are true? $f(x) = \begin{cases} \frac{x^2-4}{x-2}; if\ x \neq 2 \\ 1\ ; if\ x = 2 \end{cases}$

   I. f has a limit at $x = 2$. II. f is continuous at $x = 2$. III. f is differentiable at $x = 2$.

   A. I only B. II only C. III only D. I and II only E. I, II, and III

4. (NC – 2003) For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

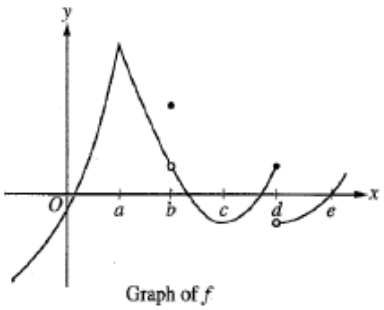
   A. $f(0) = 2$
   B. $f(x) \neq 2$ for all $x \geq 0$
   C. $f(2)$ is undefined
   D. $\lim_{x\to 2} f(x) = \infty$
   E. $\lim_{x\to\infty} f(x) = 2$

5. (NC – 2003) $\lim_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}$

   A. 4 B. 1 C. $\frac{1}{4}$ D. 0 E. -1

6. (NC – 2003) The graph of a function f is shown at right. At which value of x is f continuous, but not differentiable?

   A. a
   B. b
   C. c
   D. d
   E. e



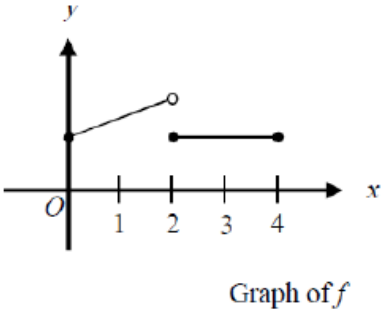
Graph of f

7. (NC – 2003) Let f be the function given below. Which of the following statements are true about f? $f(x) = \begin{cases} x+2 & x \leq 3 \\ 4x-7 & x > 3 \end{cases}$

   I. $\lim_{x\to 3} f(x)$ exists. II. f is continuous at $x = 3$. III. f is differentiable at $x = 3$.

   A. None B. I only C. II only D. I and II only E. I, II, and III

8. (C – 2008) The figure at right shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?



Graph of f

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

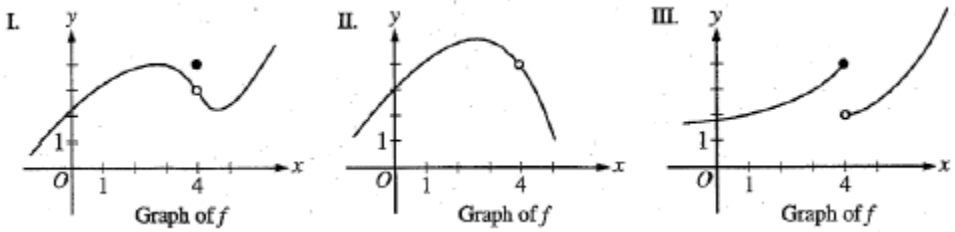
A. I only
B. II only
C. I and II only
D. I and III only
E. I, II, and III

9. (C – 2008) The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

A. For $-2 < k < 2$, $f'(k) > 0$.
B. For $-2 < k < 2$, $f'(k) > 0$.
C. For $-2 < k < 2$, $f'(k)$ exists .
D. For $-2 < k < 2$, $f'(k)$ exists but f' is not continous.
E. For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

10. (C – 2003) for which of the following does $\lim_{x \to 4} f(x)$ exist?

A. I only
B. II only
C. III only
D. I and II only
E. I and III only



Graph of f Graph of f Graph of f

11. (NC – Sample) $\lim_{x \to \pi} \frac{\cos x + \sin(2x) + 1}{x^2 - \pi^2}$

A. $\frac{1}{2\pi}$ B. $\frac{1}{\pi}$ C. 1 D. Nonexistent

12. (NC) $\lim_{x \to 0} \frac{(1+x)^6 - 1}{x}$

A. 0 B. 1 C. 6 D. ∞ E. Nonexistent

13. (NC) $\lim_{x \to 0} \frac{\cos x - 1}{x}$

A. -1
B. 0
C. 1
D. ∞
E. None of these

HW #2 – Derivatives and Tangents Review

1. (NC – 2008) If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

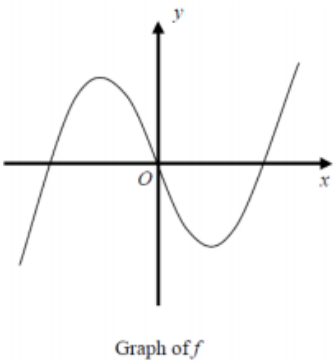
A. $6x(x^2+2)^2$ C. $(x^2+2)^2(x^2+3x-1)$ E. $-3(x-1)(x^2+2)^2$

B. $6x(x-1)(x^2+2)^2$ D. $(x^2+2)^2(7x^2-6x+2)$

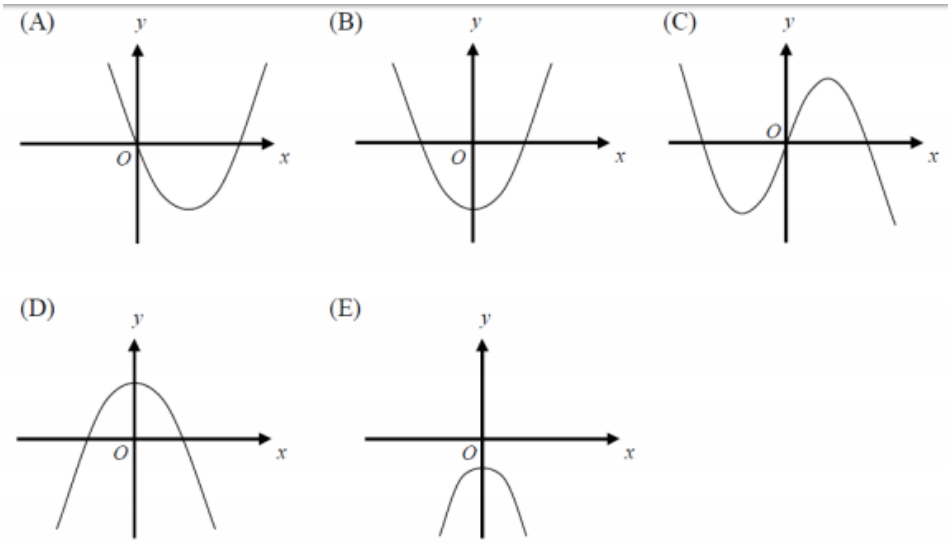
2. (NC – 2008) If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

A. $\frac{3\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $-\frac{\sqrt{3}}{2}$ D. $-\frac{3}{2}$ E. $-\frac{3\sqrt{3}}{2}$

3. (NC – 2008) The graph of a function f is shown at right. Which of the following could be the graph of the derivative of f?



Graph of f



4. (NC – 2008) If $f(x) = e^{(2/x)}$, then $f'(x) =$

A. $2e^{(2/x)} \ln x$ B. $e^{(2/x)}$ C. $e^{(-2/x^2)}$ D. $-\frac{2}{x^2}e^{(2/x)}$ E. $-2x^2e^{(2/x)}$

5. (NC – 2008) If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$

A. $\frac{2\ln x+2}{x}$ C. $2\ln x + 2$ E. $\frac{2x+2}{x}$

B. $2x\ln x + 2$ D. $2\ln x + \frac{2}{x}$

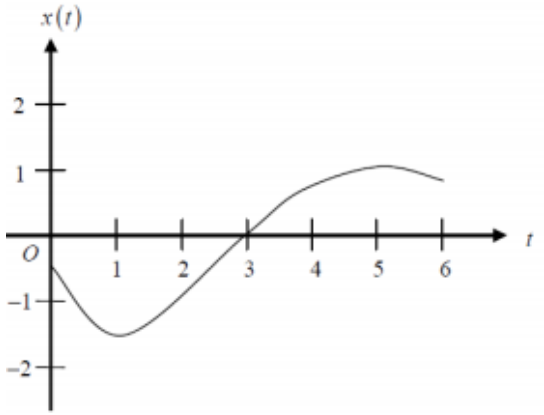
6. (NC – 2008) In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

A. -3 B. -2 C. -1 D. 0 E. 1

7. (NC – 2008) If $\sin(xy) = x$, then $\frac{dy}{dx} =$

A. $\frac{1}{\cos(xy)}$ B. $\frac{1}{x\cos(xy)}$ C. $\frac{1-\cos(xy)}{\cos(xy)}$ D. $\frac{1-y\cos(xy)}{x\cos(xy)}$ E. $\frac{y(1-\cos(xy))}{x}$

8. (NC – 2008) A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown below for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?



   A. $0 < t < 2$
   B. $1 < t < 5$
   C. $2 < t < 6$
   D. $3 < t < 5$ only
   E. $1 < t < 2$ and $5 < t < 6$

9. (NC – 2008) The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

   A. 0.4 B. 0.6 C. 0.7 D. 1.3 E. 1.4

10. (NC – 2008) Let f be the function defined below, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$? $f(x) = \begin{cases} cx + d & for\ x \le 2 \\ x^2 - cx & for\ x > 2 \end{cases}$

   A. -4 B. -2 C. 0 D. 2 E. 4

11. (NC – 2008) What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

   A. 2 B. ½ C. 0 D. -½ E. -2

12. (NC – 2008) Let f be a differentiable function such that $f(3) = 15, f(6) = 3$, $f'(3) = -8,$ and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

   A. $-\frac{1}{2}$ B. $-\frac{1}{8}$ C. $\frac{1}{6}$ D. $\frac{1}{3}$ E. Cannot be determined

13. (C – 2008) A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \ge 0$. What is the acceleration of the particle at time $t = 3$?

A. -0.914 B. 0.055 C. 5.486 D. 6.086 E. 18.087

HW #3 – Applications of Derivatives and Review

1. (NC – 2008) The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

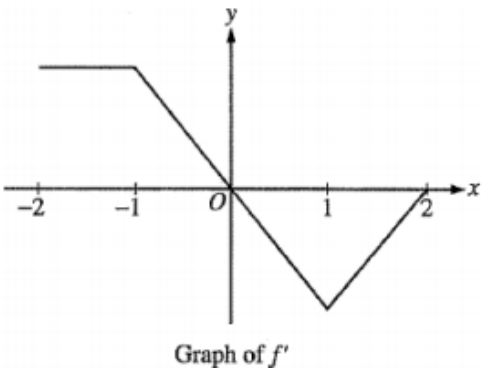
x	0	1	2	3
$f''(x)$	5	0	-7	4

   A. f is increasing on the interval $(0,2)$.
   B. f is decreasing on the interval $(0,2)$.
   C. f has a local maximum at $x = 1$.
   D. The graph of f has a point of inflection at $x = 1$.
   E. The graph of f changes concavity in the interval $(0,2)$.

2. (NC – 2008) Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x –coordinates of the points of inflection of the graph of f?

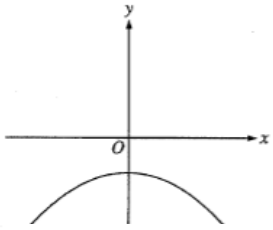
A. 0 only B. 3 only C. 0 and 6 only D. 3 and 6 only E. 0, 3, and 6

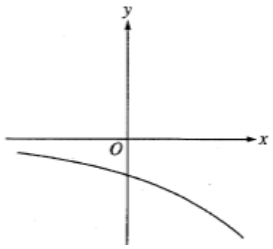
3. (NC – 2003) The graph of f', the derivative of the function f, is shown at right. Which of the following statements is true about f?
   A. f is decreasing for $-1 \le x \le 1$.
   B. f is increasing for $-2 \le x \le 0$.
   C. f is increasing for $1 \le x \le 2$.
   D. f has a local minimum at $x = 0$.
   E. f is not differentiable at $x = -1$ and $x = 1$.

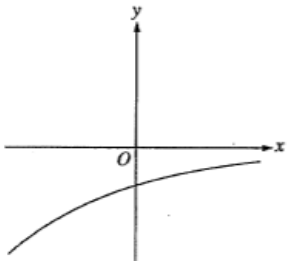


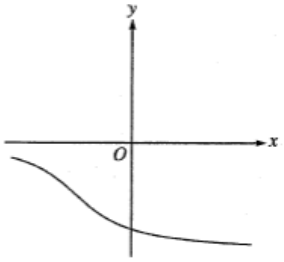
Graph of f'

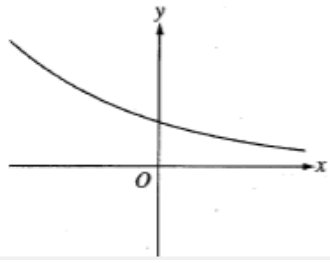
4. (NC – 2003) The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f?

(A)


(B)


(C)


(D)


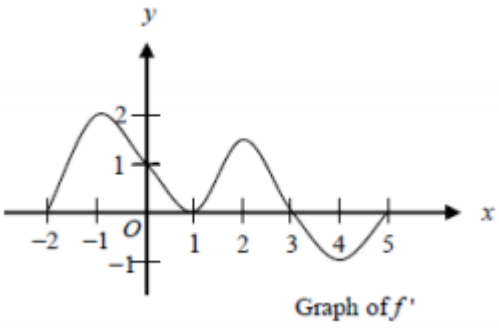
(E)


5. (NC – 2003) Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

   A. $(-\infty, -1]$ only
   B. $(-\infty, 0)$
   C. $[-1, 0)$ only
   D. $(0, \sqrt[3]{2})$
   E. $[\sqrt[3]{2}, \infty)$

6. (C – 2008) The graph of f', the derivative of f, is shown at right for $-2 \le x \le 5$. On what interval(s) is f increasing?

   A. $[-2, 1]$ only
   B. $[-2, 3]$
   C. $[3, 5]$ only
   D. $[0, 1.5]$ and $[3, 5]$
   E. $[-2, -1]$, $[1, 2]$, $[4, 5]$



7. (C – 2008) The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what interval(s) is f increasing?

   A. $1 \le x \le 1.445$
   B. $1 \le x \le 1.691$
   C. $1.445 \le x \le 1.875$
   D. $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$
   E. $0 \le x \le 1$ and $1.691 \le x \le 2$

8. (C – 2008) The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

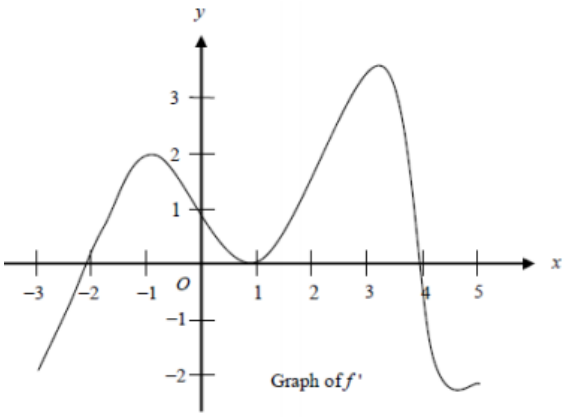
   A. 1 B. 2 C. 3 D. 4 E. 5

9. (C – 2008) The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area *S* of a sphere with radius *r* is $S = 4\pi r^2$)

   A. -108π B. -72π C. -48π D. -24π E. -16π

10. (C – 2008) The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1, x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

    A. -2 only
    B. 1 only
    C. 4 only
    D. -1 and 3 only
    E. -2, 1, and 4

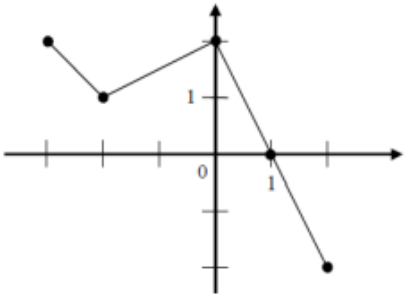


HW #4 – Integration Techniques Review

1. (NC – 2008) $\int \frac{1}{x^2} dx =$
   A. $\ln x^2 + C$
   B. $-\ln x^2 + C$
   C. $x^{-1} + C$
   D. $-x^{-1} + C$
   E. $-2x^{-3} + C$

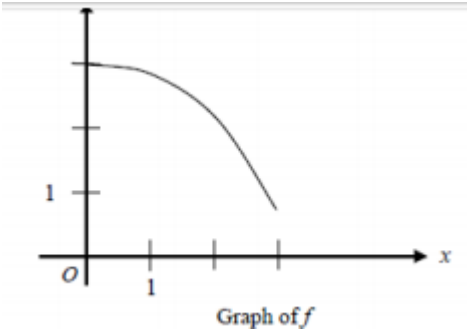
2. (NC – 2008) $\int (\sin(2x) + \cos(2x))dx =$
   A. $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$
   B. $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$
   C. $2\cos(2x) + 2\sin(2x) + C$
   D. $2\cos(2x) - 2\sin(2x) + C$
   E. $-2\cos(2x) + 2\sin(2x) + C$

3. (NC – 2008) The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)dt$, which of the following values is greatest?
   A. $g(-3)$
   B. $g(-2)$
   C. $g(0)$
   D. $g(1)$
   E. $g(2)$



Graph of f

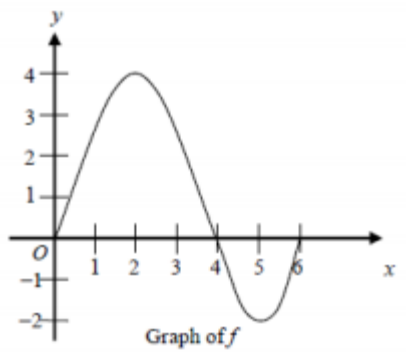
4. (NC – 2008) The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?
   A. $\int_1^3 f(x)\,dx$
   B. Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 equal subintervals
   C. Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 equal subintervals
   D. Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 equal subintervals
   E. Trapezoidal Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 equal subintervals



Graph of f

5. (NC – 2008) $\int \frac{x}{x^2-4} dx =$
   A. $\frac{-1}{4(x^2-4)^2} + C$
   B. $\frac{1}{2(x^2-4)} + C$
   C. $\frac{1}{2}\ln|x^2 - 4| + C$
   D. $2\ln|x^2 - 4| + C$
   E. $\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

6. (NC – 2008) The graph of the function f shown above has horizontal tangents at $x = 2$ and $x - 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?



   A. 2 only
   B. 4 only
   C. 2 and 5 only
   D. 2, 4, and 5
   E. 0, 4, and 6

7. (C – 2008) If $\int_{-5}^{2} f(x)dx = -17$ and $\int_{5}^{2} f(x)dx = -4$, what is the value of $\int_{-5}^{5} f(x)dx$?

   A. -21 B. -13 C. 0 D. 13 E. 21

8. (C – 2008) If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

   A. $f'(4)$
   B. $-7 + f'(4)$
   C. $\int_2^4 f(t)dt$
   D. $\int_2^4 \big(-7 + f(t)\big)dt$
   E. $-7 + \int_2^4 f(t)dt$

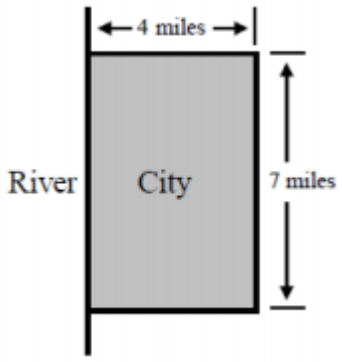
9. (C – 2008) What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 15$ and $y = x + 5$?

   A. 10.667 B. 11.833 C. 14.583 D. 21.333 E. 32

10. (C – 2008) What is the average value of $y = \frac{\cos x}{x^2+x+2}$ on the closed interval $[-1, 3]$?

    A. -0.085 B. 0.090 C. 0.183 D. 0.244 E. 0.732

11. (C – 2008) A city beside a river has a rectangular boundary as shown in the figure at right. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?



   A. $\int_0^4 f(x)\,dx$
   B. $7\int_0^4 f(x)\,dx$
   C. $28\int_0^4 f(x)\,dx$
   D. $\int_0^7 f(x)\,dx$
   E. $4\int_0^7 f(x)\,dx$

HW #5 – Particle Motion, Differential Equations, etc.

1. (NC – 2008) A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

   A. 4 B. 6 C. 9 D. 11 E. 12

2. (NC – 2008) A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

   A. $0 < t < 2$
   B. $1 < t < 5$
   C. $2 < t < 6$
   D. $3 < t < 5$ only
   E. $1 < t < 2$ and $5 < t < 6$

3. (NC – 2008) Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

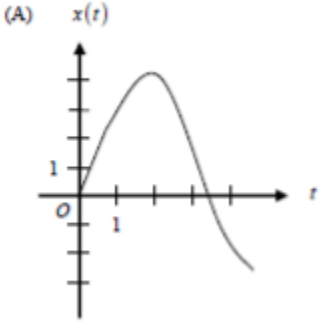
   A. $y = 2e^{-9+x^3/3}$
   B. $y = -2e^{-9+x^3/3}$
   C. $y = \sqrt{\frac{2x^3}{3}}$
   D. $y = \sqrt{\frac{2x^3}{3} - 14}$
   E. $y = -\sqrt{\frac{2x^3}{3} - 14}$

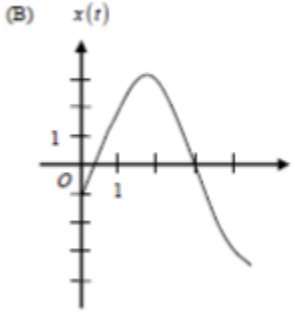
4. (NC – 2003) A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

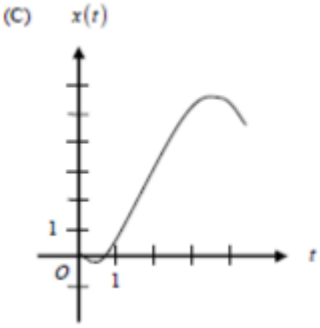
   A. $t = 1$ only
   B. $t = 3$ only
   C. $t = \frac{7}{2}$ only
   D. $t = 3$ and $t = \frac{7}{2}$
   E. $t = 3$ and $t = 4$

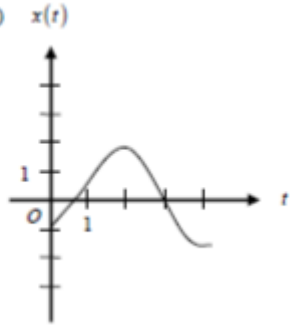
5. (C – 2008) The table gives selected values of the velocity, $v(t)$, of a particle moving along the $x - axis$. At time $t = 0$, the particle is at the origin. Which of the following could be the position, $x(t)$, of the particle for $0 \leq t \leq 4$?

t	0	1	2	3	4
$v(t)$	-1	2	3	0	-4

(A) 

(B) 

(C) 

(D) 

6. (C – 2008) An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0) = 2$ and the velocity of the object is $v(t) = \sqrt[3]{1 + t^2}$, what is the position of the object at time $t = 3$?

   A. 0.431 B. 2.154 C. 4.512 D. 6.512 E. 17.408

7. (C – 2003) A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

   A. -2.016 B. -0.677 C. 1.633 D. 1.814 E. 2.978

8. (C – 2003) The velocity, in $\frac{ft}{sec}$, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

   A. 20.086 ft/sec C. 32.809 ft/sec E. 79.342 ft/sec
   B. 26.447 ft/sec D. 40.671 ft/sec

9. (NC – 2003) A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

   A. 0.462 B. 1.609 C. 2.555 D. 2.886 E. 3.346

10. (NC – Sample) At time t, a population of bacteria grows at the rate of $5e^{0.2t} + 4t$ grams per day where t is measured in days. By how many grams has the population grown from time $t = 0$ days to $t = 10$ days?

    A. $5e^2 + 40$ B. $5e^2 + 195$ C. $25e^2 + 175$ D. $25e^2 + 375$

11. (NC – Sample) Which of the following is the solution to the differential equation $\frac{dy}{dx} = y\sec^2 x$ with initial condition $y\left(\frac{\pi}{4}\right) = -1$?

    A. $y = -e^{\tan x}$
    B. $y = -e^{(-1+\tan x)}$
    C. $y = -e^{\left(\sec^3 x - 2\sqrt{2}\right)/3}$
    D. $y = -\sqrt{2\tan x - 1}$