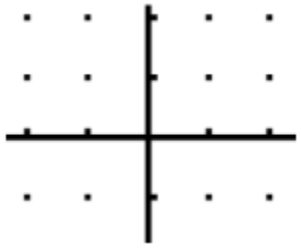
Differential Equations: Study Guide

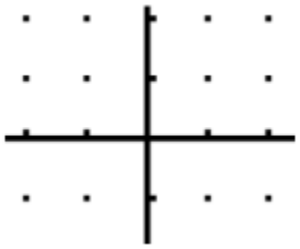
Name _____

A. Slope Fields

1. Draw a slope field for the differential equation: $\frac{dy}{dx} = x + 1$.



2. Draw a slope field for the differential equation: $\frac{dy}{dx} = \frac{x^2}{y}$



B. Euler's Method

1. Use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.
   a. $y' = x + y; y(0) = 2; n = 2; h = 0.1$
   b. $y' = 0.5x(3 - y); y(0) = 1; n = 3; h = 0.4$

C. Differential Equations

1. Under ideal conditions, air pressure decreases continuously with height above sea level at a rate proportional to the pressure at that height. If the barometer reads 30 inches at sea level and 15 inches at 18,000 feet, find the barometric pressure at 35,000 feet.

2. Find the general solution of each differential equation.
   a. $3y = y'$
   b. $(3 + x)dy = 5ydx = 0$
   c. $\frac{dy}{dx} = \frac{4x}{e^{3y}}$
   d. $\frac{dy}{dx} = 2y - 1$
   e. $y' - 2xy = 0$
   f. $y' - e^y \sin x = 0$

3. Find the particular solution of each differential equation with the given initial condition.
   a. $y' = 5y; y(0) = 12$
   b. $xy' - 3y = 0; y(2) = -3$
   c. $2xy' - \ln x^2 = 0; y(1) = 2$
   d. $xe^{x^2} + y\,y' = 0\,; y(0) = 1$

4. $\frac{dy}{dx} = \frac{2x}{y}; y(0) = 2$
   a. Use integration to find the particular solution of the differential equation.
   b. Use your solution to evaluate $y(1)$
   c. Use Euler's method with a step size of 0.5 to estimate $y(1)$.
   d. Is your answer from part (c) an overestimation or an underestimation? Explain why.

5. Find the logistic equation that passes through the given point: $\frac{dy}{dt} = y\left(1 - \frac{P}{100}\right), (0,4)$

6. A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the environment can support no more than 4000 elk. The growth rate of the elk population p is $\frac{dP}{dt} = kp\left(1 - \frac{p}{4000}\right), 40 \leq p \leq 4000$
   a. Write a model for the elk population in terms of t.
   b. Use the model to estimate the elk population after 15 years.
   c. Find the limit of the model as $t \to \infty$.
7. A conservation organization releases 25 Florida panthers into a game preserve. After 2 years, there are 39 panthers in the preserve. The Florida preserve has a carrying capacity of 200 panthers.
   a. Write a logistic equation that models the population of panthers in the preserve.
   b. Find the population after 5 years.
   c. When will the population reach 100?
   d. Write a logistic differential equation that models the growth rate of the panther population. Then repeat part (b) using Euler's Method with a step size of h = 1. Compare your approximation with the exact answer.
   e. When is the panther population growing most rapidly?

8. Free Response: No Calculator (except for part C)

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$

(A) On the axes provided, sketch a slope field for the given differential equation.

(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$.

(C) Find the particular solution $y = g(x)$ to the differential equation with the initial condition $g(1) = 1$. Use your solution to find $g(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (B) to the actual value of $g(1.2)$ found in part (C). Was your estimate from part (B) an underestimate or an overestimate? Use your slope field to explain why.