* Chain Rule and Implicit Differentiation Review *

1. A. $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(-1) = f'(g(-1)) \cdot g'(-1)$

$h'(-1) = f'(2) \cdot g'(-1)$

$h'(-1) = 4 \cdot -3 = \boxed{-12}$

B. $h(x) = [f(x)]^2$

$h'(x) = 2f(x)\, f'(x)$

$h'(2) = 2f(2)\, f'(2)$

$h'(2) = 2\,(0)\,(4) = \boxed{0}$

C. $h(x) = [g(f(x))]^3$

$h'(x) = 3[g(f(x))]^2 \cdot g'(f(x)) \cdot f'(x)$

$h'(-1) = 3[g(f(-1))]^2 \cdot g'(f(-1)) \cdot f'(-1)$

$h'(-1) = 3[g(2)]^2 \cdot g'(2) \cdot 3$

$h'(-1) = 3[1]^2 \cdot -5 \cdot 3 = \boxed{-45}$

4. $f(x) = \left(\frac{1+x^2}{1+x^6}\right)^{11}$

$f'(x) = 11\left(\frac{1+x^2}{1+x^6}\right)^{10}\left[\frac{(1+x^6)(2x)-(1+x^2)(6x^5)}{(1+x^6)^2}\right]$

$f'(x) = \frac{11\,(1+x^2)^{10}(2x+2x^7-6x^5-6x^7)}{(1+x^6)^{12}}$

$\boxed{f'(x) = \frac{11\,(1+x^2)^{10}(2x-6x^5-4x^7)}{(1+x^6)^{12}}}$

5. $5x^3 = -3xy + 2$

$15x^2 = -3x\frac{dy}{dx} + y(-3)$

$3x\frac{dy}{dx} = -3y - 15x^2$

$\frac{dy}{dx} = \frac{-3y-15x^2}{3x}$

$\boxed{\frac{dy}{dx} = \frac{-y-5x^2}{x}}$

2. $y = \cos(1-x)$

$y' = -\sin(1-x) \cdot -1$

$\boxed{y' = \sin(1-x)}$

3. $y = \sec\left(\sqrt{x^3+x}\right)$

$y = \sec\,(x^3+x)^{1/2}$

$y' = \sec\sqrt{x^3+x}\,\tan\sqrt{x^3+x} \cdot \frac{1}{2}(x^3+x)^{-1/2}\,(3x^2+1)$

$\boxed{y' = \frac{(3x^2+1)\sec\sqrt{x^3+x}\,\tan\sqrt{x^3+x}}{2\sqrt{x^3+x}}}$

6. $3x^2y^2 = 4x^2 - 4xy$

$3x^2 \cdot 2y\frac{dy}{dx} + y^2(6x) = 8x - 4x\frac{dy}{dx} + y(-4)$

$6x^2y\frac{dy}{dx} + 6xy^2 = 8x - 4x\frac{dy}{dx} - 4y$

$6x^2y\frac{dy}{dx} + 4x\frac{dy}{dx} = 8x - 4y - 6xy^2$

$\frac{dy}{dx}(6x^2y + 4x) = 8x - 4y - 6xy^2$

$\frac{dy}{dx} = \frac{8x - 4y - 6xy^2}{6x^2y + 4x}$

$\boxed{\frac{dy}{dx} = \frac{4x - 2y - 3xy^2}{3x^2y + 2x}}$

7. $4y^2 + 2 = 3x^2$

$8y\frac{dy}{dx} = 6x$

$\frac{dy}{dx} = \frac{3x}{4y}$

$\frac{d^2y}{dx^2} = \frac{4y(3) - 3x(4\frac{dy}{dx})}{(4y)^2}$

$\frac{d^2y}{dx^2} = \frac{12y - 12x\frac{dy}{dx}}{16y^2}$

$\frac{d^2y}{dx^2} = \frac{12y - 12x(\frac{3x}{4y})}{16y^2} = \frac{12y - \frac{36x^2}{4y}}{16y^2} = \frac{\frac{48y^2 - 36x^2}{4y^2}}{16y^2} = \frac{48y^2 - 36x^2}{4y^2} \cdot \frac{1}{16y^2}$

OK

$\boxed{\frac{d^2y}{dx^2} = \frac{48y^2 - 36x^2}{64y^4}} = \boxed{\frac{12y^2 - 9x^2}{16y^4}} \Rightarrow \frac{-3(-4y^2 + 3x^2)}{16y^4} = \frac{-3(2)}{16y^4} = \boxed{\frac{-3}{8y^4}}$

OK

since in original problem $4y^2 + 2 = 3x^2$, $2 = 3x^2 - 4y^2$