AP Calculus: Chapter 4 Integration Study Guide

1. Find the general solution of the differential equation. Use the given point to find the particular solution of the equation. $\frac{dy}{dx} = 2\sqrt{x}, (4,12)$
2. Find each indefinite integral.

   a. $\int (5 - x)dx$ b. $\int \left(\frac{x^2 + x + 1}{\sqrt{x}} \right) dx$ c. $\int \left(\frac{\cos x}{1 - \cos^2 x} \right) dx$

3. Evaluate the sum: $\sum_{k=3}^{9} 2k - 5 =$
4. Use properties of summation to evaluate the limit. $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{4i^2(i-1)}{n^4} =$
5. Use the limit process to find the area of the region between the graph of the function and the x-axis over the interval. $y = 2x - x^3, [0,1]$
6. Approximate the area of the region bounded by the curve $y = \sqrt{x} + 2$ and the x-axis on the interval $[0,2]$.
   a. Use a right hand sum and 4 subintervals of equal width.
   b. Use a midpoint sum and two subintervals of equal width.
7. Set up a limit that would evaluate the definite integral: $\int_1^3 3x^2 dx$
8. Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, evaluate

   a. $\int_0^6 f(x)dx$ b. $\int_6^3 f(x)dx$ c. $\int_3^3 f(x)dx$ d. $\int_3^6 -5f(x)dx$

9. Evaluate $\int_1^8 \sqrt{\frac{2}{x}}dx$ using the Fundamental Theorem of Calculus.
10. Find the value(s) of ***c*** guaranteed by the Mean Value Theorem for Integrals for the function over the given integral. $f(x) = \frac{5}{x^3}, [2,6]$.
11. Find the average value of the function over the given interval.
    $f(x) = \cos x, \left[0, \frac{\pi}{2}\right]$

12. Find the derivative of each.

a. $F(x)=\int_{4}^{x}\sqrt{t}\,dt$

b. $F(x)=\int_{4}^{2x}(x^2+4)\,dt$

13. Find the indefinite integral.

a. $\int\sqrt[3]{(1-2x^2)}(4x)\,dx$

b. $\int x^2\sqrt{x^3+2}\,dx$

c. $\int(x+1)\sqrt{2-x}\,dx$

d. $\int\dfrac{\cos^3\theta}{2-2\sin^2\theta}dx$

14. Find the definite integral.

a. $\int_{-2}^{4}x^2(x^3+8)^2\,dx$

b. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(x+\cos x)\,dx$

15. Use the differential equation and the given point to find an equation for the function. $\dfrac{dy}{dx}=\dfrac{-48}{(3x+5)^3}$ and (-1,3).

16. The velocity of a particle moving along the x-axis is given by $v(t)=4-t^2, t>0$. What is the average velocity of the particle from time $t=1$ to $t=4$?

17. On a certain day, the rate at which clothing is donated to a thrift store is modeled by the function R, where $R(t)$ is measured in pounds per hour and t is the number of hours since the store opened. Using a trapezoidal sum with the three subintervals indicated in the table, what is the approximate number of pounds of clothing donated in the first 7 hours since the center opened?

t (hours)	0	2	3	7
R(t) (pounds per hour)	45	29	12	34

18. An object moves along a straight line so that at any time t its acceleration is given by $a(t)=8t$. At time $t=0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t=4$?

19. To help restore land damaged by Hurricane Matthew, sod is being added to a landscape at a rate of $s(t)=40+12\sin(0.2t)$ tons per hour where t is measured in hours since 7:00 AM. How many tons of sod have been added to the landscape over the 4 hour period from 7:00 AM to 11:00 AM?