8.2 Notes: Arithmetic Sequences and Partial Sums

Arithmetic Sequences, also known as a discrete linear function, is a sequence for which consecutive terms have a common difference, $d$.

Determine whether or not the sequence is arithmetic. If it is, find the common difference.

<table>
<thead>
<tr>
<th></th>
<th>1. 5, 8, 11, 14, 17,...</th>
<th>2. 1, 4, 9, 16, 25,...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[3, 3, 3, 3]</td>
<td>[3, 7, 9]</td>
</tr>
<tr>
<td></td>
<td>\text{yes - arithmetic}</td>
<td>\text{Not arithmetic}</td>
</tr>
<tr>
<td></td>
<td>(d = 3)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>3. (\frac{1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, ...})</th>
<th>4. (1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{5}{6}, \frac{7}{6}, \frac{8}{6}, \frac{10}{6}, ...)</td>
<td>(\frac{12}{12}, \frac{6}{12}, \frac{1}{12}, \frac{1}{12}, ...)</td>
</tr>
<tr>
<td></td>
<td>(d = \frac{1}{2})</td>
<td>\text{Not arithmetic}</td>
</tr>
</tbody>
</table>

Writing an explicit formula/rule for an arithmetic sequence $a_n$.

Fill in the missing terms from the sequence:

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n$</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td>22</td>
<td>25</td>
</tr>
</tbody>
</table>

Expanded:

\[a_2 = 4 + 3\]
\[a_3 = 4 + 3 + 3\]
\[a_4 = 4 + 3 + 3 + 3\]

Condensed:

\[a_2 = a_1 + 3(1)\]
\[a_3 = a_1 + 3(2)\]
\[a_4 = a_1 + 3(3)\]

Arithmetic Explicit Rule:

\[a_n = a_1 + d(n-1)\]

Write an explicit rule for the given sequence. Then answer any additional questions. Assume $n \geq 1$.

5. 5, 12, 19, 26,...

\[d = 7\]

\[a_n = a_1 + d(n-1)\]

\[a_n = 5 + 7(n-1)\]

\[a_n = 7n - 2\]

6. Find an explicit formula for $a_n$ for the arithmetic sequence with the following terms:

\[a_3 = 19\] and \[a_6 = 27\].

* Not consecutive!

Find $d$ using the constant rate of change: slope.

\[d = \frac{27 - 19}{5 - 3} = \frac{8}{2} = 4\]

\[a_n = a_1 + 4(n-1)\]

\[19 = a_1 + 4(2-1)\]

\[19 = a_1 + 8\]

\[a_n = 11 + 4(n-1)\]
7. \( d = -4 \)
\( a_n = 29 - 4(n-1) \)
\( a_9 = 29 - 4(9-1) \)
\( a_9 = -4n + 33 \)

9. Find the first five terms of the arithmetic sequence where \( a_8 = 25 \) and \( a_{12} = 41 \).
\( d = \frac{41-25}{12-8} = \frac{16}{4} = 4 \)
\( a_n = a_1 + 4(n-1) \)
\( 25 = a_1 + 4(8-1) \)
\( 25 = a_1 + 28 \)
\( a_1 = -3 \)

10. Find the 10\textsuperscript{th} term of the arithmetic sequence whose first two terms are 8 and 20.
\( d = 12 \)
\( a_n = a_1 + 12(n-1) \)
\( a_{10} = 8 + 12(10-1) \)
\( a_{10} = 8 + 12(9) \)
\( a_{10} = 116 \)

**Arithmetic Series**

Find the sum of:

| 40 + 37 + 34 + 31 + 28 + 25 + 22 |

The _______ of a finite arithmetic sequence with \( n \) terms (\( n^{th} \) partial sum) can be found by:

\[ S_n = \frac{n}{2} (a_1 + a_n) \]

Where \( n = \text{number of terms} \), \( a_1 = \text{first term} \), and \( a_n = \text{last term} \).

Find the sum of the finite arithmetic sequence.

11. Sum of integers from 1 to 35.
\( 1 + 2 + 3 + 4 + \ldots + 35 \)

Arithmetic rule \( d = 1 \)

35 terms
\( a_1 = 1 \)
\( a_{35} = 35 \)
\[ S_{35} = \frac{35}{2}(1 + 35) \]
\[ S_{35} = 590 \]

13. 50\textsuperscript{th} partial sum of the arithmetic sequence -6, -2, 2, 6, ...
\( d = 4 \)
\[ S_{50} = \frac{50}{2}(-6 + a_{50}) \]
\[ S_{50} = \frac{50}{2}(-6 + 190) \]
\[ S_{50} = 4600 \]

12. Sum of odd integers from 1 to 57
\( 1 + 3 + 5 + \ldots + 57 \)

How many terms:
\( a_n = a_1 + d(n-1) \)
\( 57 = 1 + 2(n-1) \)
\( 56 = 2(n-1) \)
\( n = 28 \)
\[ S_{28} = \frac{28}{2}(1 + 57) \]
\[ S_{28} = 841 \]

14. Determine the seating capacity of an auditorium with 30 rows of seats if there are 20 seats in the first row, 22 in the second, 24 in the third, and so on.

\[ 20 + 22 + 24 + \ldots + 60 \]
\( d = 2 \)
\[ q_{30} = 20 + 2(30-1) \]
\[ q_{30} = 78 \]
\[ S_{30} = \frac{30}{2}(20 + 78) \]
\[ S_{30} = 1470 \text{ seats} \]

15. \( \sum_{n=1}^{100} (2 + 3n) \)

Arithmetic rule
\( a_1 = 2 + 3(1) = 5 \)
\( a_{100} = 2 + 3(100) = 302 \)

100 terms in the series.
\[ S_{100} = \frac{100}{2}(5 + 302) \]
\[ S_{100} = 15350 \]

16. \( \sum_{n=21}^{100} (2 + 3n) \)

Arithmetic rule
\( a_{21} = 2 + 3(21) = 65 \)
\( a_{100} = 2 + 3(100) = 302 \)

80 terms.
\[ S_{80} = \frac{80}{2}(65 + 302) \]
\[ S_{80} = 14680 \]