8.1 Notes: Sequences and Series—Day 1

A **Sequence** is a function whose **DOMAIN** is a set of consecutive integers. If a domain is NOT SPECIFIED it is understood that the domain starts with **1**. The values in the **RANGE** are called the **terms** of the sequence.

<table>
<thead>
<tr>
<th>Domain</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>...</th>
<th>...</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>$a_n$</td>
</tr>
</tbody>
</table>

A **finite sequence** has a limited number of terms. An example would be: 1, 2, 4, 8, 16

a) How many terms are in this sequence? **5**

b) What is $a_3$? **4**

c) Write a rule for finding the nth term. $a_n = 2^{(n-1)}$

An **infinite sequence** continues without stopping. The set of natural numbers is an example of an infinite sequence. What are the natural numbers? **1, 2, 3, 4, 5, ...**

a) What is $a_5$? **5**

Instead of using function notation, sequences are usually written using subscript notation.

<table>
<thead>
<tr>
<th>Write the first five terms of the sequence.</th>
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<th>Find the 3rd, 4th and 5th term of the sequence.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n = 2n + 1$</td>
<td>$a_n = 2 - (-1)^n$</td>
<td>$a_n = \frac{2 + (-1)^n}{n}$</td>
</tr>
<tr>
<td>$a_1 = 2(1) + 1 = 3$</td>
<td>$a_1 = 2 - (-1)^1 = 3$</td>
<td>$a_3 = \frac{2 + (-1)^3}{3} = \frac{1}{3}$</td>
</tr>
<tr>
<td>$a_2 = 2(2) + 1 = 5$</td>
<td>$a_2 = 2 - (-1)^2 = 1$</td>
<td>$a_4 = \frac{2 + (-1)^4}{4} = \frac{3}{4}$</td>
</tr>
<tr>
<td>$a_3 = 2(3) + 1 = 7$</td>
<td>$a_3 = 2 - (-1)^3 = 3$</td>
<td>$a_5 = \frac{2 + (-1)^5}{5} = \frac{1}{5}$</td>
</tr>
<tr>
<td>$a_4 = 2(4) + 1 = 9$</td>
<td>$a_4 = 2 - (-1)^4 = 1$</td>
<td>$\frac{1}{3}, \frac{3}{4}, \frac{1}{5}$</td>
</tr>
<tr>
<td>$a_5 = 2(5) + 1 = 11$</td>
<td>$a_5 = 2 - (-1)^5 = 3$</td>
<td></td>
</tr>
</tbody>
</table>

| Write an expression for the apparent nth term of the sequence. (Assume n begins with 1). |
|-----------------------------------|-----------------------------------------------|
| 4. $a_n = 2, 4, 6, 8, \ldots$    | 5. $a_n = 1, 3, 5, 7$                        |
| What is the rule? $a_n = 2n$     | What is the rule? $a_n = 2n - 1$              |
| What is $a_7$? $a_7 = 2(7) = 14$ | What is $a_8$? **Does not exist... only 4 terms** |
Write an expression for the apparent \( n^{th} \) term of the sequence. (Assume \( n \) begins with 1).

6. \( a_n = 1, 4, 9, 16 \)

\[ a_n = n^2 \]

7. \( a_n = 2, 5, 10, 17, \ldots \)

\[ a_n = n^2 + 1 \]

\[ \text{Each term is 1 more than \( n^2 \)} \]

8. \( a_n = 1, 2, 7, 14, 23, \ldots \)

\[ a_n = n^2 - 2 \]

\[ \text{Each term is 2 less than \( n^2 \)} \]

9. \( a_n = 1, 2, -7, 14, -23, \ldots \)

\[ a_n = (-1)^n \cdot (n^2 - 2) \]

\[ \text{(-1)}^n \text{ alternate signs!} \]

When a sequence is built using PREVIOUS TERMS the sequence is said to be defined recursively.

Fill in the missing terms:

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( a_6 )</th>
<th>( a_7 )</th>
<th>( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>6</td>
<td>24</td>
<td>120</td>
<td>720</td>
<td>5040</td>
<td></td>
</tr>
</tbody>
</table>

To find the rule:

\[ a_2 = a_1 \cdot 2 \]

\[ a_3 = a_2 \cdot 3 \]

\[ a_4 = a_3 \cdot 4 \]

\[ a_n = a_{n-1} \cdot n \]

10. Consider the sequence 1, 1, 2, 3, 5, 8, 13, 21, ...

Describe the pattern in words.

*Starting with the 3rd term...

Each term is found by adding the 2 previous terms...*

Write a recursive formula to define this sequence.

\[ a_3 = a_2 + a_1 \]

\[ a_4 = a_3 + a_2 \]

\[ a_n = a_{n-1} + a_{n-2} \text{ where } n \geq 3 \]

What is this very famous sequence of numbers called? **Fibonacci Sequence**

11. Write the first five terms of the sequence.

\[ a_{k+1} = \frac{1}{2} a_k; \quad a_1 = 32 \]

\[ a_2 = \frac{1}{2} (32) = 16 \]

Note, \( k = 1 \)

\[ a_3 = \frac{1}{2} (16) = 8 \]

\( k = 2 \)

\[ a_4 = \frac{1}{2} (8) = 4 \]

\( k = 3 \)

\[ a_5 = \frac{1}{2} (4) = 2 \]

\( k = 4 \)

32, 16, 8, 4, 2

Write an expression for the apparent \( n^{th} \) term of the sequence.

\[ a_n = 32 \left( \frac{1}{2} \right)^{(n-1)} \]