5.9 p. 393 (1-21 odds, 31)

1. $\int \frac{5}{\sqrt{9-x^2}}\,dx$

$5\int \frac{dx}{\sqrt{3^2-x^2}}$ $a=3$ $u=x$

$\boxed{5\arcsin\left(\frac{x}{3}\right)+C}$

3. $\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}}\,dx$

$\int_0^{1/6} \frac{dx}{\sqrt{1-(3x)^2}}$ $a=1$ $u=3x$ $du=3dx$

$\frac{1}{3}\int_0^{1/6} \frac{3dx}{\sqrt{1-(3x)^2}}$ $x=0,\ u=0$ $x=1/6\ u=\frac{1}{2}$

$\frac{1}{3}\int_0^{1/2} \frac{du}{\sqrt{1-u^2}} = \frac{1}{3}\arcsin u\Big]_0^{1/2}$

$\frac{1}{3}[\arcsin\frac{1}{2}-\arcsin 0]$

$\frac{1}{3}[\pi/6-0] = \boxed{\pi/18}$

5. $\int \frac{7}{16+x^2}\,dx = 7\int \frac{dx}{4^2+x^2}$ $a=4$ $u=x$

$\boxed{\frac{7}{4}\arctan\frac{x}{4}+C}$

7. $\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx = \int_0^{\sqrt{3}/2} \frac{dx}{1+(2x)^2}$ $u=2x$ $a=1$ $du=2dx$

$\frac{1}{2}\int_0^{\sqrt{3}/2} \frac{2dx}{1+(2x)^2}$ If $x=0,\ u=0$ $x=\sqrt{3}/2\ u=\sqrt{3}$

$\frac{1}{2}\int_0^{\sqrt{3}} \frac{du}{1+u^2} = \frac{1}{2}\arctan u\Big]_0^{\sqrt{3}}$

$\frac{1}{2}[\arctan\sqrt{3}-\arctan 0] = \frac{1}{2}[\frac{\pi}{3}-0] = \boxed{\frac{\pi}{6}}$

9. $\int \frac{1}{x\sqrt{4x^2-1}}\,dx = \int \frac{dx}{x\sqrt{(2x)^2-1}}$ $u=2x$ $a=1$ $du=2dx$

$\int \frac{2dx}{2x\sqrt{(2x)^2-1}} = \int \frac{du}{u\sqrt{(u)^2-1}} =$

$\frac{1}{1}\operatorname{arcsec}|u|+C$

$\boxed{\frac{1}{1}\operatorname{arcsec}|2x|+C}$

13. $\int \frac{1}{\sqrt{1-(x+1)^2}}\,dx$ $u=x+1$ $du=dx$ $a=1$

$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$

$\boxed{\arcsin(x+1)+C}$

15. $\int \frac{t}{\sqrt{1-t^4}}\,dt = \int \frac{t}{\sqrt{1-(t^2)^2}}\,dt$

$u=t^2$ $du=2t\,dt$ $\frac{1}{2}\int \frac{2t}{\sqrt{1-(t^2)^2}}\,dt$

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\arcsin u + C$

$\boxed{\frac{1}{2}\arcsin(t^2)+C}$

17. $\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$ $u = \arcsin x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$\int_0^{\pi/4} u\,du$ $x=0,\ u=\arcsin 0 = 0$

$x = \frac{1}{\sqrt{2}},\ u = \arcsin\left(\frac{1}{\sqrt{2}}\right) = \pi/4$

$\frac{u^2}{2}\Big]_0^{\pi/4} = \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - 0^2\right] = \boxed{\frac{\pi^2}{32}}$

19. $\int_{-1/2}^{0} \frac{x}{\sqrt{1-x^2}}\,dx$ $u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int_{-1/2}^{0} \frac{-2x\,dx}{\sqrt{1-x^2}}$ $x = -\frac{1}{2},\ u = 1 - \frac{1}{4} = 3/4$

$x=0,\ u = 1-0 = 1$

$-\frac{1}{2}\int_{3/4}^{1} u^{-1/2}\,du$

$-\frac{1}{2}\left[2u^{1/2}\right]_{3/4}^{1} = -\sqrt{u}\Big]_{3/4}^{1} = -\sqrt{1} - -\sqrt{\frac{3}{4}}$

$\boxed{-1 + \frac{\sqrt{3}}{2} \approx -0.134}$

21. $\int \frac{e^{2x}}{4+e^{4x}}\,dx$ $u = e^{2x}$ $a = 2$

$du = 2e^{2x}\,dx$

$\frac{1}{2}\int \frac{2e^{2x}\,dx}{2^2 + (e^{2x})^2} = \frac{1}{2}\int \frac{du}{2^2+u^2}$

$\frac{1}{2}\cdot\frac{1}{2}\arctan\frac{u}{2} + C$

$\boxed{\frac{1}{4}\arctan\frac{e^{2x}}{2} + C}$

31. $\int_0^2 \frac{dx}{x^2-2x+2}$

$x^2 - 2x + 2$

$x^2 - 2x + 1 + 2 - 1$

$x^2 - 2x + 1 + 1$

$(x-1)^2 + 1$

$\int_0^2 \frac{dx}{(x-1)^2+1} = \int_0^2 \frac{dx}{1+(x-1)^2}$

$\arctan(x-1)\Big]_0^2$

$\arctan 1 - \arctan -1$

$\frac{\pi}{4} - \frac{-\pi}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$