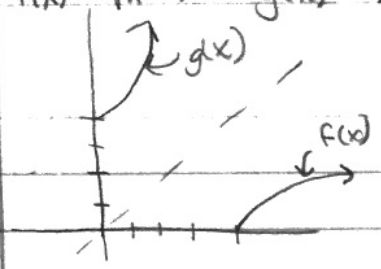
5.3 p. 338 (5, 13, 15, 17, 25, 29, 33, 71, 73, 75)

5. $f(x) = \sqrt{x-4}$ $g(x) = x^2+4$, $x \geq 0$



$f(g(x)) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$

$g(f(x)) = (\sqrt{x-4})^2 + 4 = x-4+4 = x$

13. yes, HLT passes.
inverse exists

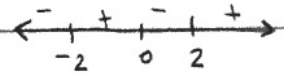
15. no, HLT does not pass.
inverse does not exist

17. $h(s) = \frac{1}{s-2} - 3$

yes, function is one-to-one

25. $f(x) = \frac{x^4}{4} - 2x^2$

$f'(x) = x^3 - 4x = x(x^2-4)$



f(x) is not strictly monotonic
∴ inverse of f does not exist

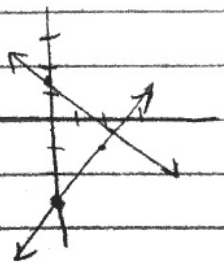
29. $f(x) = 2x-3$

$x = 2y-3$

$x+3 = 2y$

$\frac{1}{2}(x+3) = y$

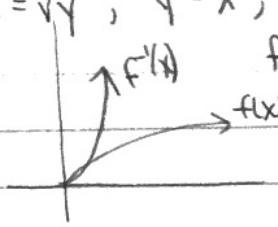
$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$



33. $f(x) = \sqrt{x}$

$x = \sqrt{y}$, $y = x^2$, $x \geq 0$

$f^{-1}(x) = x^2$, $x \geq 0$



71. $f(x) = x^3 + 2x - 1$ $a = 2$

$f(g(x)) = x$

$f'(g(x)) \cdot g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$ $f(x) = 2$ when $x = 1$

$\therefore f^{-1}(2) = 1$

$(f^{-1})'(2) = \frac{1}{f'(1)}$ $f'(x) = 3x^2 + 2$

$f'(1) = 5$

$\boxed{(f^{-1})'(2) = \frac{1}{5}}$

73. $f(x) = \sin x$ $a = \frac{1}{2}$ $\sin x = \frac{1}{2}$ when $x = \frac{\pi}{6}$ so $f^{-1}(\frac{1}{2}) = \pi/6$

$(f^{-1})'(\frac{1}{2}) = \frac{1}{f'(f^{-1}(\frac{1}{2}))}$

$(f^{-1})'(\frac{1}{2}) = \frac{1}{f'(\pi/6)}$ $f'(x) = \cos x$

$(f^{-1})'(\frac{1}{2}) = \frac{1}{\cos \pi/6} = \frac{1}{\frac{\sqrt{3}}{2}} = \boxed{\frac{2}{\sqrt{3}}}$

75. $f(x) = x^3 - \frac{4}{x}$ $a = 6$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(6))}$$

$$f(x) = x^3 - \frac{4}{x} = 6$$

When $x = 2$, $f(x) = 6$

$2^3 - \frac{4}{2} = 6$,

so $f^{-1}(6) = 2$

$$(f^{-1})'(x) = \frac{1}{f'(2)}$$

$$\boxed{(f^{-1})'(x) = \frac{1}{13}}$$

$f(x) = x^3 - 4x^{-1}$

$f'(x) = 3x^2 + 4x^{-2}$

$f'(x) = 3x^2 + \frac{4}{x^2}$

$f'(2) = 3(2)^2 + \frac{4}{2^2} = 13$