5.1 pp 321-323 (21, 25, 31, 33, 37, 39, 41, 45, 49, 55, 57, 63, 65, 71, 73, 77)

21. $\ln\left(\frac{xy}{z}\right)$

$\ln(xy) - \ln z$

$\boxed{\ln x + \ln y - \ln z}$

25. $\ln\left(\frac{x^2-1}{x^3}\right)^3$

$3\ln\left(\frac{x^2-1}{x^3}\right)$

$3\ln(x^2-1) - 3\ln x^3$

$\boxed{3\ln(x^2-1) - 9\ln x}$

31. $\frac{1}{3}\left[2\ln(x+3) + \ln x - \ln(x^2-1)\right]$

$\frac{1}{3}\left[\ln(x+3)^2 + \ln x - \ln(x^2-1)\right]$

$\frac{1}{3}\left[\ln\left(x(x+3)^2\right) - \ln(x^2-1)\right]$

$\frac{1}{3}\ln\left(\frac{x(x+3)^2}{x^2-1}\right)$

$\ln\left(\frac{x(x+3)^2}{x^2-1}\right)^{1/3} = \boxed{\ln\sqrt{\frac{x(x+3)^2}{x^2-1}}}$

33. $2\ln 3 - \frac{1}{2}\ln(x^2+1)$

$\ln 3^2 - \ln(x^2+1)^{1/2}$

$\ln 9 - \ln\sqrt{x^2+1}$

$\boxed{\ln\frac{9}{\sqrt{x^2+1}}}$

37. $\lim_{x\to 3^+} \ln(x-3) = \boxed{-\infty}$

39. $\lim_{x\to 2^-} \ln[x^2(3-x)] = \ln[4(1)] = \boxed{\ln 4}$

41. $y = \ln x^3 = 3\ln x$

$y' = \frac{3}{x}$, $(1,0)$

$y' = \frac{3}{1} = 3$

$y - 0 = 3(x-1)$

$\boxed{y = 3x - 3}$

45. $g(x) = \ln(x^2)$

$g'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

$\boxed{g'(x) = \frac{2}{x}}$

49. $y = \ln\left(x\sqrt{x^2-1}\right)$

Option #1

$y' = \frac{1}{x\sqrt{x^2-1}} \cdot \left[x \cdot \frac{1}{2}(x^2-1)^{-1/2}(2x) + (x^2-1)^{1/2}\right]$

$y' = \frac{x^2}{(x^2-1)^{1/2}} + (x^2-1)^{1/2}$

$x(x^2-1)^{1/2}$

$y' = \frac{x^2}{(x^2-1)^{1/2}} + \frac{(x^2-1)}{(x^2-1)^{1/2}} = \frac{2x^2-1}{(x^2-1)^{1/2}}$

$x(x^2-1)^{1/2}$

$y' = \frac{2x^2-1}{(x^2-1)^{1/2}} \cdot \frac{1}{x(x^2-1)^{1/2}} = \boxed{\frac{2x^2-1}{x(x^2-1)}}$

$y = \ln\left(x\sqrt{x^2-1}\right) = \ln x + \ln\sqrt{x^2-1}$

$y = \ln x + \frac{1}{2}\ln(x^2-1)$

Option #2

$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x^2-1} \cdot 2x$

$y' = \frac{1}{x} + \frac{x}{x^2-1} = \frac{x^2-1}{x^2-1} \cdot \frac{1}{x} + \frac{x}{x^2-1} \cdot \frac{x}{x}$

$y' = \frac{x^2-1}{x(x^2-1)} + \frac{x^2}{x(x^2-1)}$

$\boxed{y' = \frac{2x^2-1}{x(x^2-1)}}$

55. $y = \ln(\ln x^2)$

$y' = \frac{1}{\ln x^2} \cdot \frac{1}{x^2} \cdot 2x$

$y' = \frac{2}{x \ln x^2} = \frac{2}{x \cdot 2\ln x} = \boxed{\frac{1}{x \ln x}}$

57. $y = \ln\sqrt{\frac{x+1}{x-1}}$

$y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$y = \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x-1)$

$y' = \frac{1}{2}\cdot\frac{1}{x+1} - \frac{1}{2}\cdot\frac{1}{x-1}$

$y' = \frac{1}{2(x+1)} - \frac{1}{2(x-1)}$

$y' = \frac{x-1}{2(x+1)(x-1)} - \frac{(x+1)}{2(x+1)(x-1)}$

$y' = \frac{-2}{2(x+1)(x-1)} = \frac{-1}{(x+1)(x-1)} = \boxed{\frac{-1}{x^2-1} = \frac{1}{1-x^2}}$

Either answer!

63. $y = \ln|\sin x|$

$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \boxed{\cot x}$

65. $y = \ln\left|\frac{\cos x}{\cos x - 1}\right|$

$y = \ln|\cos x| - \ln|\cos x - 1|$

$y' = \frac{1}{\cos x} \cdot -\sin x - \frac{1}{\cos x - 1} \cdot -\sin x$

$y' = \frac{-\sin x}{\cos x} + \frac{\sin x}{\cos x - 1}$

$\boxed{y' = -\tan x + \frac{\sin x}{\cos x - 1}}$

71. $f(x) = 3x^2 - \ln x \quad (1,3)$

$f'(x) = 6x - \frac{1}{x}$

$f'(1) = 6(1) - \frac{1}{1} = 5$

$\boxed{y - 3 = 5(x-1)}$

73. $x^2 - 3\ln y + y^2 = 10$

multiply by "y" →

$2x + \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2xy + 3\frac{dy}{dx} + 2y^2\frac{dy}{dx} = 0$

$3\frac{dy}{dx} + 2y^2\frac{dy}{dx} = -2xy$

$\frac{dy}{dx}(3 + 2y^2) = -2xy$

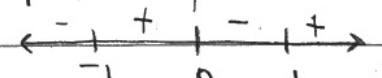
$\boxed{\frac{dy}{dx} = \frac{-2xy}{3+2y^2}}$

77. $y = \frac{1}{2}x^2 - \ln x$

$y' = x - \frac{1}{x} = \frac{x^2-1}{x}$

$y' = 0$ when $x^2 - 1 = 0$, $x = \pm 1$

y' DNE when $x = 0$



Domain of y: $(0, \infty)$

$\therefore$ $f'(x)$ changes from neg. to pos. at $x=1$, so f has a rel. min at $x=1$.

$y'' = \frac{x(2x) - (x^2-1)}{x^2} = \frac{2x^2 - x^2 + 1}{x^2}$

$y'' = \frac{x^2+1}{x^2}$ $y'' \neq 0$, y'' DNE when $x=0$ but $x=0$ not in domain.

$\therefore$ no points of inflection