

4.5B Integration by Substitution

pp. 297-299 (57, 59, 65, 69, 71, 75, 83, 85, 89)

57. $\int x\sqrt{x+2} dx$ $u=x+2$ $x=u-2$
 $du=dx$

$$\int (u-2)u^{1/2} du$$

$$\int (u^{3/2} - 2u^{1/2}) du$$

$$\frac{2u^{5/2}}{5} - \frac{2 \cdot 2u^{3/2}}{3} + C$$

$$\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

69. $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

$u=2x+1$

$du=2dx$

$x=0, u=1$

$x=4, u=9$

$$\int_0^4 (2x+1)^{-1/2} dx$$

$$\frac{1}{2} \int_0^4 (2x+1)^{1/2} (2dx)$$

59. $\int x^2\sqrt{1-x} dx$ $u=1-x$ $du=-dx$
 $x=1-u$

$$-\int (1-u)^2 u^{1/2} du$$

$$-\int (1-2u+u^2)u^{1/2} du$$

$$-\int (u^{1/2} - 2u^{3/2} + u^{5/2}) du$$

$$-\left[\frac{2u^{3/2}}{3} - \frac{2 \cdot 2u^{5/2}}{5} + \frac{2u^{7/2}}{7} \right] + C$$

$$-\frac{2}{3}u^{3/2} + \frac{4}{5}u^{5/2} - \frac{2}{7}u^{7/2} + C$$

$$-\frac{2}{3}(1-x)^{3/2} + \frac{4}{5}(1-x)^{5/2} - \frac{2}{7}(1-x)^{7/2} + C$$

$$\frac{1}{2} \int_1^9 u^{-1/2} du = \frac{1}{2} \left[u^{1/2} \right]_1^9 =$$

$$u^{1/2} \Big|_1^9 = 9^{1/2} - 1^{1/2} = 3 - 1 = \boxed{2}$$

71. $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

$u=1+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2} dx$

$du = \frac{1}{2\sqrt{x}} dx$

$dx = 2\sqrt{x} du$

$$2 \int_1^9 \frac{1}{2\sqrt{x}(1+\sqrt{x})^2} dx$$

$$+ \int_2^4 u^{-2} du$$

$x=1, u=2$

$x=9, u=4$

65. $\int_{-1}^1 x(x^2+1)^3 dx$ $u=x^2+1$ $x=-1, u=2$
 $du=2x dx$ $x=1, u=2$

$$\frac{1}{2} \int_{-1}^1 2x(x^2+1)^3 dx$$

$$\frac{1}{2} \int_2^2 u^3 du = \boxed{0}$$

$$2 \left[\frac{u^{-1}}{-1} \right]_2^4 = 2 \left[\frac{-1}{u} \right]_2^4 =$$

$$2 \left[\frac{-1}{4} - \frac{-1}{2} \right] = \boxed{\frac{1}{2}}$$

$$75. \int_0^{\pi/2} \cos\left(\frac{2x}{3}\right) dx \quad u = \frac{2x}{3}$$

$$du = \frac{2}{3} dx$$

$$\frac{3}{2} \int_0^{\pi/2} \cos\left(\frac{2x}{3}\right) \left(\frac{2}{3} dx\right) \quad x=0, u=0$$

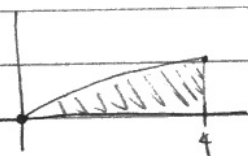
$$x = \frac{\pi}{2}, u = \frac{\pi}{3}$$

$$\frac{3}{2} \int_0^{\pi/3} \cos u \, du$$

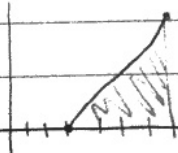
$$\frac{3}{2} [\sin u]_0^{\pi/3} = \frac{3}{2} \left[\sin \frac{\pi}{3} - \sin 0 \right]$$

$$\frac{3}{2} \left[\frac{\sqrt{3}}{2} - 0 \right] = \boxed{\frac{3\sqrt{3}}{4}}$$

$$83. \int_0^4 \frac{x}{\sqrt{2x+1}} dx = \boxed{\frac{4}{3}}$$



$$85. \int_3^7 x\sqrt{x-3} \, dx = \boxed{28.8}$$



$$89. \int (2x-1)^2 dx$$

Option 1:

$$\int (4x^2 - 4x + 1) dx$$

$$\frac{4x^3}{3} - \frac{4x^2}{2} + x + C$$

$$\boxed{\frac{4}{3}x^3 - 2x^2 + x + C}$$

Option 2:

$$u = 2x-1 \quad \frac{1}{2} \int (2x-1)^2 \cdot 2 dx$$

$$du = 2 dx \quad \frac{1}{2} \int u^2 du$$

$$\frac{1}{3} u^3 + C$$

$$(2x-1)(2x-1)(2x-1)$$

$$(4x^2 - 4x + 1)(2x-1)$$

$$8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$$

$$8x^3 - 12x^2 + 6x - 1$$

$$\frac{1}{6} (u^3) + C$$

$$\frac{1}{6} (2x-1)^3 + C$$

$$\frac{1}{6} (8x^3 - 12x^2 + 6x - 1) + C$$

$$\frac{4}{3}x^3 - 2x^2 + x - \frac{1}{6} + C = \boxed{\frac{4}{3}x^3 - 2x^2 + x + C}$$