

### 4.5A Integration by Substitution

pp. 297-299 (9-37 EOO, 41, 45)

7.  $\int (1+2x)^4 (2) dx$   $u=1+2x$   
 $du=2dx$

$$\int u^4 du = \frac{u^5}{5} + C$$

$$\boxed{\frac{1}{5}(1+2x)^5 + C}$$

21.  $\int \frac{x}{\sqrt{1-x^2}} dx$

$$\int x(1-x^2)^{-1/2} dx \quad u=1-x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} \int (1-x^2)^{-1/2} dx$$

$$-\frac{1}{2} \int u^{-1/2} du$$

$$-\frac{1}{2} \cdot 2 u^{1/2} + C = -u^{1/2} + C$$

$$\boxed{-(1-x^2)^{1/2} + C}$$

9.  $\int \sqrt{9-x^2} (-2x) dx$   $u=9-x^2$   
 $du = -2x dx$

$$\int u^{1/2} du = \frac{2u^{3/2}}{3} + C$$

$$\boxed{\frac{2}{3}(9-x^2)^{3/2} + C}$$

25.  $\int (1+\frac{1}{t})^3 (\frac{1}{t^2}) dt$

$$u = 1+t^{-1}$$

$$du = -t^{-2} dt = -\frac{1}{t^2} dt$$

$$-\int (1+\frac{1}{t})^3 (\frac{1}{t^2}) dt$$

$$-\int u^3 du = -\frac{u^4}{4} + C = \boxed{-\frac{1}{4}(1+\frac{1}{t})^4 + C}$$

13.  $\int x^2 (x^3-1)^4 dx$   $u=x^3-1$   
 $du = 3x^2 dx$

$$\frac{1}{3} \int 3x^2 (x^3-1)^4 dx$$

$$\frac{1}{3} \int u^4 du = \frac{1}{3} \cdot \frac{u^5}{5} + C$$

$$\boxed{\frac{1}{15}(x^3-1)^5 + C}$$

29.  $\int \frac{x^2+3x+7}{\sqrt{x}} dx = \int (x^{1/2} + \frac{3x}{x^{1/2}} + \frac{7}{x^{1/2}}) dx$

$$\int (x^{3/2} + 3x^{1/2} + 7x^{-1/2}) dx =$$

17.  $\int 5x \sqrt[3]{1-x^2} dx$   $u=1-x^2$

$$5 \int x \sqrt[3]{1-x^2} dx \quad du = -2x dx$$

$$\frac{1}{2} \cdot 5 \int -2x \sqrt[3]{1-x^2} dx$$

$$-\frac{5}{2} \int u^{1/3} du = -\frac{5}{2} \cdot \frac{3u^{4/3}}{4} + C$$

$$-\frac{15}{8} u^{4/3} + C$$

$$\boxed{-\frac{15}{8}(1-x^2)^{4/3} + C}$$

$$\frac{2}{5} x^{5/2} + \frac{2 \cdot 3}{3} x^{3/2} + \frac{2 \cdot 7}{1} x^{1/2} + C$$

$$\boxed{\frac{2}{5} x^{5/2} + 2x^{3/2} + 14x^{1/2} + C}$$

$$33. \int (9-y)\sqrt{y} dy$$

$$\int (9y^{1/2} - y^{3/2}) dy$$

$$2 \cdot \frac{9^{3/2}}{3} - \frac{2}{5} y^{5/2} + C$$

$$\boxed{6y^{3/2} - \frac{2}{5}y^{5/2} + C}$$

$$37. \frac{dy}{dx} = \frac{x+1}{(x^2+2x-3)^2}$$

$$y = \int \frac{x+1}{(x^2+2x-3)^2} dx \quad u = x^2+2x-3$$

$$du = (2x+2)dx$$

$$y = \frac{1}{2} \int \frac{(2x+2)dx}{(x^2+2x-3)^2}$$

$$y = \frac{1}{2} \int u^{-2} du$$

$$y = \frac{1}{2} \frac{u^{-1}}{-1} + C$$

$$y = \frac{-1}{2(u)} + C$$

$$\boxed{y = \frac{-1}{2(x^2+2x-3)} + C}$$

$$45. \int \frac{1}{\theta^2} \cos\left(\frac{1}{\theta}\right) d\theta$$

$$u = \frac{1}{\theta} = \theta^{-1}$$

$$du = -1\theta^{-2}d\theta = \frac{-1}{\theta^2}d\theta$$

$$- \int \frac{1}{\theta^2} \cos\left(\frac{1}{\theta}\right) d\theta$$

$$- \int \cos u du$$

$$- \sin u + C$$

$$\boxed{-\sin\left(\frac{1}{\theta}\right) + C}$$

$$41. \int \pi \sin \pi x dx \quad u = \pi x$$

$$\int \sin u du \quad du = \pi dx$$

$$- \cos u + C$$

$$\boxed{-\cos(\pi x) + C}$$