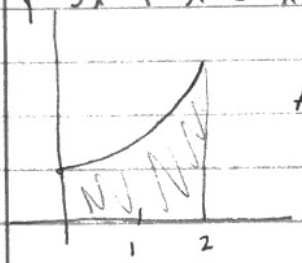
4.4B FTOC

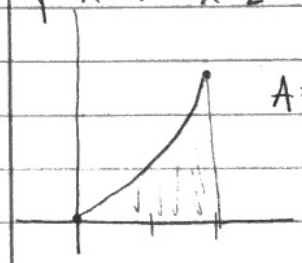
Pp. 284 - 287 (41-51 odds, 75-91 odds)

41. $y = 3x^2+1$ $x=0$ $x=2$ $y=0$



$$A = \int_0^2 (3x^2+1)\,dx = \frac{3x^3}{3} + x \Big]_0^2 = (2^3-2) - (0^3+0) = \boxed{6}$$

43. $y = x^3+x$ $x=2$ $y=0$



$$A = \int_0^2 (x^3+x)\,dx = \frac{x^4}{4} + \frac{x^2}{2} \Big]_0^2 = \left(\frac{2^4}{4} - \frac{2^2}{2}\right) - \left(\frac{0^4}{4} - \frac{0^2}{2}\right) = \boxed{2}$$

45. $f(x) = x - 2\sqrt{x}$ $[0,2]$

$$\int_0^2 [x - 2x^{1/2}]\,dx = f(c)(2-0)$$

$$\frac{x^2}{2} - \frac{2 \cdot 2x^{3/2}}{3} \Big]_0^2 = f(c) \cdot 2$$

$$\frac{x^2}{2} - \frac{4x^{3/2}}{3} \Big]_0^2 = f(c) \cdot 2$$

$$\left(\frac{2^2}{2} - \frac{4 \cdot 2^{3/2}}{3}\right) - \left(\frac{0^2}{2} - \frac{4 \cdot 0^{3/2}}{3}\right) = f(c) \cdot 2$$

$$2 - \frac{4}{3}\sqrt{8} = f(c) \cdot 2$$

$$\frac{1}{2}\left(2 - \frac{4}{3}\sqrt{8}\right) = f(c)$$

$$1 - \frac{2}{3}\sqrt{8} = f(c)$$

$$\boxed{c = 0.438,\ 1.791}$$

*I used my calculator and looked for intersections here.

47. $f(x) = 2\sec^2 x$ $[-\pi/4, \pi/4]$

$$\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx = f(c) \cdot \left(\frac{\pi}{4} - \frac{-\pi}{4}\right)$$

$$2\tan x \Big]_{-\pi/4}^{\pi/4} = f(c)\left(\frac{\pi}{2}\right)$$

$$2\left[\tan\frac{\pi}{4} - \tan\frac{-1}{4}\right] = f(c) \cdot \frac{\pi}{2}$$

$$2[1 - (-1)] = f(c) \cdot \frac{\pi}{2}$$

$$4 = f(c) \cdot \frac{\pi}{2}$$

$$\frac{8}{\pi} = f(c)$$

So: $\frac{8}{\pi} = 2\sec^2 x$

$$\frac{4}{\pi} = \sec^2 x$$

$$\boxed{x = \pm 0.482}$$

49. $f(x) = 4 - x^2 \quad [-2, 2]$

$\frac{1}{b-a}\int_a^b f(x)\,dx \qquad \frac{1}{4}\int_{-2}^{2}(4-x^2)\,dx = \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$

$\frac{1}{4}\left[\left(4\cdot 2 - \frac{2^3}{3}\right) - \left(4\cdot -2 - \frac{(-2)^3}{3}\right)\right]$

$\frac{1}{4}\left[\frac{16}{3} - \frac{-16}{3}\right]$

$\boxed{\frac{8}{3}}$

$f(x) = \frac{8}{3} = 4 - x^2$

$-\frac{4}{3} = -x^2 \Rightarrow x^2 = \frac{4}{3} \Rightarrow x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}} = \boxed{\pm 1.155}$

51. $f(x) = \sin x \quad [0, \pi]$

$\frac{1}{\pi}\int_0^{\pi} \sin x\,dx = \frac{1}{\pi}(-\cos x)\Big]_0^{\pi} = \frac{1}{\pi}\left[(-\cos\pi) - (-\cos 0)\right] = \frac{1}{\pi}\left[1 + 1\right] = \frac{2}{\pi}$

$f(x) = \frac{2}{\pi} = \sin x$ $\boxed{x = 0.690, 2.451}$

75. $F(x) = \int_0^x (t+2)\,dt$

$F(x) = \frac{t^2}{2} + 2t\Big]_0^x$

$F(x) = \left(\frac{x^2}{2} + 2x\right) - \left(\frac{0^2}{2} + 2\cdot 0\right)$

$\boxed{F(x) = \frac{1}{2}x^2 + 2x}$

$\boxed{F'(x) = x + 2}$

79. $F(x) = \int_{\pi/4}^{x} \sec^2 x\,dx$

$F(x) = \tan x\Big]_{\pi/4}^{x} = \tan x - \tan\frac{\pi}{4}$

$\boxed{f(x) = \tan x - 1}$

$\boxed{F'(x) = \sec^2 x}$

81. $F(x) = \int_{-2}^{x} (t^2 - 2t)\,dt$

$\boxed{F'(x) = x^2 - 2x}$

83. $F(x) = \int_{-1}^{x} \sqrt{t^4 + 1}\,dt$

$\boxed{F'(x) = \sqrt{x^4 + 1}}$

85. $F(x) = \int_{0}^{x} t\cos t\,dt$

$\boxed{F'(x) = x\cos x}$

87. $F(x) = \int_{x}^{x+2} (4t+1)\,dt$

$F(x) = \int_{x}^{a} (4t+1)\,dt + \int_{a}^{x+2} (4t+1)\,dt$

$F(x) = -\int_{a}^{x} (4t+1)\,dt + \int_{a}^{x+2} (4t+1)\,dt$

$F'(x) = -(4x+1) + 4(x+2)+1$

$F'(x) = -4x - 1 + 4x + 8 + 1$

$\boxed{F'(x) = 8}$

89. $F(x) = \int_{0}^{\sin x} \sqrt{t}\,dt$

$F'(x) = \sqrt{\sin x} \cdot \cos x$

$\boxed{F'(x) = \cos x \sqrt{\sin x}}$

91. $F(x) = \int_{0}^{x^3} \sin t^2\,dt$

$F'(x) = \sin(x^3)^2 \cdot 3x^2$

$\boxed{F'(x) = 3x^2 \sin x^6}$