4.2B Area

pp. 261-264 (23, 25, 27, 49, 51, 53 + 55 set up only)

23. Upper Sum: $(1)\cdot f(1) + (1)\cdot f(2) + 1\cdot f(3) + 1\cdot f(4)$

$1(3) + 1(4) + 1(4.5) + 1(5) = \boxed{16.5}$

Lower Sum: $(1)f(0) + (1)f(1) + (1)f(2) + (1)f(3)$

$0 + 1\cdot 3 + 1\cdot 4 + 1\cdot 4.5 = \boxed{11.5}$

25. Upper Sum: $1\cdot f(1) + 1\cdot f(3) + 1\cdot f(4) = 3+3+5 = \boxed{11}$

Lower Sum: $1\cdot f(2) + 1\cdot f(2) + 1\cdot f(3) = 2+2+3 = \boxed{7}$

27. Upper Sum: $\frac{1}{4}\cdot f(\frac{1}{4}) + \frac{1}{4}f(\frac{1}{2}) + \frac{1}{4}f(\frac{3}{4}) + \frac{1}{4}f(1)$

$\frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{\sqrt{2}} + \frac{1}{4}\cdot\sqrt{\frac{3}{4}} + \frac{1}{4}\cdot 1 =$

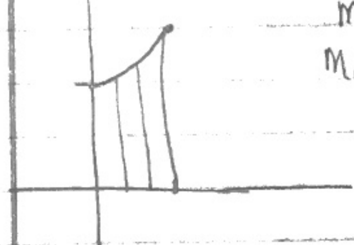
Lower Sum: $\frac{1}{4}f(0) + \frac{1}{4}f(\frac{1}{4}) + \frac{1}{4}f(\frac{1}{2}) + \frac{1}{4}f(\frac{3}{4})$

$\frac{1}{4}(0) + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{\sqrt{2}} + \frac{1}{4}\cdot\sqrt{\frac{3}{4}} =$

49. $y = x^2 + 2\ [0,1]$ $\Delta x = \frac{1}{n}$ $A = \lim\limits_{n\to\infty}\sum\limits_{n=1}^{\infty} f(\frac{i}{n})\frac{1}{n}$

$m_i = 0 + i(\frac{1}{n})$

$M_i = i/n$



$A = \lim\limits_{n\to\infty}\sum\limits_{n=1}^{\infty}\left[\left(\frac{i}{n}\right)^2 + 2\right]\frac{1}{n} = \lim\limits_{n\to\infty}\sum\limits_{n=1}^{\infty}\frac{i^2}{n^3} + \frac{2}{n}$

$\lim\limits_{n\to\infty}\left[\sum\limits_{n=1}^{\infty}\frac{i^2}{n^3} + \sum\limits_{n=1}^{\infty}\frac{2}{n}\right]$

$\lim\limits_{n\to\infty}\frac{1}{n^3}\sum\limits_{n=1}^{\infty} i^2 + \frac{1}{n}\sum\limits_{n=1}^{\infty} 2$

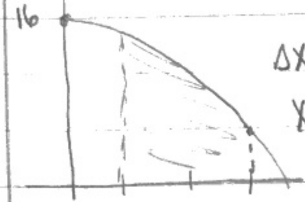
$\lim\limits_{n\to\infty}\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1}{n}(2n)$

$\lim\limits_{n\to\infty}\frac{(n^2+n)(2n+1)}{6n^3} + 2$

$\lim\limits_{n\to\infty}\frac{2n^3+2n^2+n^2+n}{6n^3} + 2 = \lim\limits_{n\to\infty}\frac{2n^3+3n^2+n}{6n^3} + 2 = \frac{1}{3} + 2$

$\boxed{\frac{7}{3}}$

51. $y = 16 - x^2 \quad [1,3]$



$$A = \lim_{n\to\infty} \sum_{n=1}^{\infty} f\left(1+\frac{2i}{n}\right)\frac{2}{n}$$

$\Delta x = \frac{2}{n}$

$x_i = 1 + \frac{2i}{n}$

$$A = \lim_{n\to\infty} \sum_{n=1}^{\infty} \left[16 - \left(1+\frac{2i}{n}\right)^2\right]\frac{2}{n}$$

$$A = \lim_{n\to\infty} \sum_{n=1}^{\infty} \left[16 - \left(1 + \frac{4i}{n} + \frac{4i^2}{n^2}\right)\right]\frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{n=1}^{\infty} \left[15 - \frac{4i}{n} - \frac{4i^2}{n^2}\right]\frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{n=1}^{\infty} \left[\frac{30}{n} - \frac{8i}{n^2} - \frac{8i^2}{n^3}\right]$$

$$\lim_{n\to\infty} \left[\sum_{n=1}^{\infty} \frac{30}{n} - \sum_{n=1}^{\infty} \frac{8i}{n^2} - \sum_{n=1}^{\infty} \frac{8i^2}{n^3}\right]$$

$$\lim_{n\to\infty} \left[\frac{30n}{n} - \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right]$$

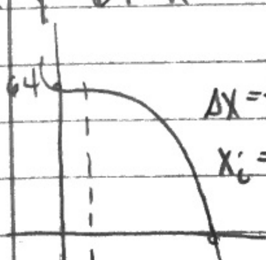
$$\lim_{n\to\infty} \left[30 - \frac{8n^2+8n}{2n^2} - \frac{8(n^2+n)(2n+1)}{6n^3}\right]$$

$$\lim_{n\to\infty} \left[30 - \frac{8n^2+8n}{2n^2} - \frac{8(2n^3+n^2+2n^2+n)}{6n^3}\right]$$

$$\lim_{n\to\infty} \left[30 - \frac{8n^2+8n}{2n^2} - \frac{16n^3+24n^2+8n}{6n^3}\right] = 30 - 4 - \frac{8}{3}$$

$$26 - 8/3 = \boxed{\frac{70}{3}}$$

53. $y = 64 - x^3 \quad [1,4]$

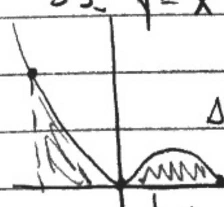


$\Delta x = \frac{3}{n}$

$x_i = 1 + \frac{3i}{n}$

$$\boxed{A = \lim_{n\to\infty} \sum_{n=1}^{\infty} f\left(1+\frac{3i}{n}\right)\frac{3}{n}}$$

55. $y = x^2 - x^3 \quad [-1, 1]$



$\Delta x = \frac{2}{n} \qquad x_i = -1 + \frac{2i}{n}$

$$\boxed{A = \lim_{n\to\infty} \sum_{n=1}^{\infty} f\left(-1+\frac{2i}{n}\right)\frac{2}{n}}$$