4.2 A Area

pp. 261-264 (1, 5, 9, 15, 17, 19, 31, 33, 35, 41)

1. $\sum_{i=1}^{5} (2i+1)$

$(2\cdot1+1)+(2\cdot2+1)+(2\cdot3+1)+(2\cdot4+1)+(2\cdot5+1)$

$3+5+7+9+11=\boxed{35}$

5. $\sum_{k=1}^{4} c = \boxed{4c}$

9. $[5(\frac{1}{8})+3]+[5(\frac{2}{8})+3] + \cdots [5(\frac{8}{8})+3]$

$\boxed{\sum_{i=1}^{8} 5(\frac{i}{8})+3}$

15. $\sum_{i=1}^{20} 2i = 20(20+1) = \boxed{420}$

$\sum_{i=1}^{20} 2i = 2\sum_{i=1}^{20} i = 2\left[\frac{(20)(20+1)}{2}\right] \boxed{420}$

$\sum_{i=1}^{n} 2i = 2\sum_{i=1}^{n} i = 2\left[\frac{n(n+1)}{2}\right] = n(n+1)$

17. $\sum_{i=1}^{20} (i-1)^2 = \sum_{i=1}^{20} (i^2-2i+1) = \frac{(20)(21)(41)}{6} - 2\left[\frac{20\cdot21}{2}\right] + 20 = \boxed{2470}$

$\sum_{i=1}^{n} (i-1)^2 = \sum_{i=1}^{n} (i^2-2i++1) = \sum_{i=1}^{n} i^2 - 2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$

$\frac{n(n+1)(2n+1)}{6} \quad 2\left[\frac{n(n+1)}{2}\right] + n$

19. $\sum_{i=1}^{15} i(i-1)^2$

$$\sum_{i=1}^{15} i(i^2-2i+1) = \sum_{i=1}^{15} i^3-2i^2+i = \overset{14{,}400}{\frac{15^2(16)^2}{4}} - \overset{-2480}{2\left[\frac{15\cdot 16\cdot 31}{6}\right]} + \overset{+120}{\frac{15\cdot 16}{2}} = \boxed{12{,}040}$$

$$\sum_{i=1}^{n} i^3-2i^2+i = \frac{n^2(n+1)^2}{4} - 2\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{n(n+1)}{2}$$

35. $\sum_{i=1}^{n} \frac{2i+1}{n^2} = \frac{1}{n^2}\sum_{i=1}^{n} 2i+1 = \frac{1}{n^2}\left[2\left(\frac{n(n+1)}{2}\right)+n\right] = \frac{1}{n^2}\left[n^2+n+n\right] =$

$\frac{1}{n^2}(n^2+2n) = 1+\frac{2}{n}$

$n=10$, $\sum_{i=1}^{10} \frac{2i+1}{n^2} = 1+\frac{2}{10} = 1.2$

$n=100$, $S = 1+\frac{2}{100} = 1.02$

$n=1000$, $S = 1+\frac{2}{1000} = 1.002$

39. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{16i}{n^2} = \lim_{n\to\infty} \frac{16}{n^2}\left(\frac{n(n+1)}{2}\right) =$

$\lim_{n\to\infty} \frac{16n(n+1)}{2n^2} = \boxed{8}$

31. $S(n) = \frac{81}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]$

$\lim_{n\to\infty} \frac{81n^2(n+1)^2}{4n^4} = \boxed{\frac{81}{4}}$

33. $S(n) = \frac{18}{n^2}\left[\frac{n(n+1)}{2}\right]$

$\lim_{n\to\infty} \frac{18n(n+1)}{2n^2} = \boxed{9}$

41. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2 = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i^2-2i+1)$

$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6} - \frac{2n(n+1)}{2} + n\right]$

$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{(n^2+n)(2n+1)}{6} - n^2 - n + n\right]$

$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3+2n^2+n^2+n}{6} - n^2\right]$

$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3+3n^2+n}{6} - \frac{6n^2}{6}\right]$

$\lim_{n\to\infty} \frac{1}{n^3}\left[\frac{2n^3-3n^2+n}{6}\right]$

$\lim_{n\to\infty} \frac{2n^3-3n^2+n}{6n^3} = \boxed{\frac{1}{3}}$