4.1 Antiderivatives and Indefinite Integration

pp. 249–251 (15–39 odds, 49, 51, 55–61 odds)

15. $\int (x+3)\,dx = \boxed{\frac{x^2}{2} + 3x + C}$

17. $\int (2x - 3x^2)\,dx = \frac{2x^2}{2} - \frac{3x^3}{3} + C$
$\boxed{x^2 - x^3 + C}$

19. $\int (x^3+2)\,dx = \boxed{\frac{x^4}{4} + 2x + C}$

21. $\int (x^{3/2} + 2x + 1)\,dx =$
$2\frac{x^{5/2}}{5} + \frac{2x^2}{2} + x + C$
$\boxed{\frac{2}{5}x^{5/2} + x^2 + x + C}$

23. $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx =$
$3\frac{x^{5/3}}{5} + C = \boxed{\frac{3}{5}x^{5/3} + C}$

25. $\int \left(\frac{1}{x^3}\right)dx = \int x^{-3}\,dx =$
$\frac{x^{-2}}{-2} + C$ $\boxed{\frac{-1}{2x^2} + C}$

27. $\int \left(\frac{x^2+x+1}{\sqrt{x}}\right)dx$
$\int (x^{3/2} + x^{1/2} + x^{-1/2})\,dx =$
$2\frac{x^{5/2}}{5} + \frac{2x^{3/2}}{3} + \frac{2x^{1/2}}{1} + C$
$\boxed{\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C}$

29. $\int (x+1)(3x-2)\,dx$
$\int (3x^2 - 2x + 3x - 2)\,dx$
$\int (3x^2 + x - 2)\,dx =$
$\frac{3x^3}{3} + \frac{x^2}{2} - 2x + C$
$\boxed{x^3 + \frac{1}{2}x^2 - 2x + C}$

31. $\int y^2\sqrt{y}\,dy = \int y^2 y^{1/2}\,dy$
$\int y^{5/2}\,dy = 2\frac{y^{7/2}}{7} + C$
$\boxed{\frac{2}{7}y^{7/2} + C}$

33. $\int dx = \boxed{x + C}$

35. $\int (2\sin x + 3\cos x)\,dx$
$\boxed{-2\cos x + 3\sin x + C}$

37. $\int (1 - \csc t \cot t)\,dt$
$\boxed{x + \csc t + C}$

39. $\int (\sec^2\theta - \sin\theta)\,d\theta$
$\boxed{\tan\theta + \cos\theta + C}$

49. $\frac{dy}{dx} = 2x - 1$ $(1, 1)$
$y = \int (2x-1)\,dx$
$y = 2\frac{x^2}{2} - x + C$
$y = x^2 - x + C$
$1 = 1^2 - 1 + C$
$1 = C$
$\boxed{y = x^2 - x + 1}$

51. $\frac{dy}{dx} = \cos x$ $(0, 4)$
$y = \int \cos x\,dx$
$y = \sin x + C$
$4 = \sin(0) + C$
$4 = C$
$\boxed{y = \sin x + 4}$

55. $f'(x) = 4x \quad f(0) = 6$

$f(x) = \int 4x\,dx$

$f(x) = \frac{4x^2}{2} + C$

$f(x) = 2x^2 + C$

$6 = 2(0)^2 + C$

$C = 6$

$\boxed{f(x) = 2x^2 + C}$

57. $h'(t) = 8t^3 + 5 \quad h(1) = -4$

$h(t) = \int (8t^3 + 5)\,dt$

$h(t) = 8\frac{t^4}{4} + 5t + C$

$h(t) = 2t^4 + 5t + C$

$-4 = 2(1)^4 + 5(1) + C$

$-4 = 7 + C \quad C = -11$

$\boxed{h(t) = 2t^4 + 5t - 11}$

59. $f''(x) = 2 \quad f'(2) = 5 \quad f(2) = 10$

$f'(x) = \int 2\,dx$

$f'(x) = 2x + C$

$5 = 2(2) + C$

$C = 1$

$f'(x) = 2x + 1$

$f(x) = \int (2x + 1)\,dx$

$f(x) = 2\frac{x^2}{2} + x + C$

$f(x) = x^2 + x + C$

$10 = 2^2 + 2 + C$

$C = 4$

$\boxed{f(x) = x^2 + x + 4}$

61. $f''(x) = x^{-3/2} \quad f'(4) = 2 \quad f(0) = 0$

$f'(x) = \int x^{-3/2}\,dx$

$f'(x) = \frac{x^{-1/2}}{-1/2} + C$

$f'(x) = -2x^{-1/2} + C$

$f'(x) = \frac{-2}{\sqrt{x}} + C$

$2 = \frac{-2}{\sqrt{4}} + C$

$2 = -1 + C, \quad C = 3$

$f'(x) = -2x^{-1/2} + 3$

$f(x) = \int (-2x^{-1/2} + 3)\,dx$

$f(x) = \frac{-2x^{1/2}}{1/2} + 3x + C$

$f(x) = -4x^{1/2} + 3x + C$

$0 = -4(0)^{1/2} + 3(0) + C$

$C = 0$

$\boxed{f(x) = -4x^{1/2} + 3x}$