3.9 HW

1. $f(x) = x^{100}$ $x = 1$

$f(1) = 1^{100} = 1$

$f'(x) = 100x^{99}$

$f'(1) = 100 \cdot 1^{99} = 100$

$y - 1 = 100(x-1)$

$\boxed{y = 1 + 100(x-1)}$

$y(1.002) = 1 + 100(1.002 - 1)$

$y = 1 + 0.2 = \boxed{1.02}$

2. $f(x) = \sqrt[3]{1+x}$ $x = 0$

$f(0) = \sqrt[3]{1} = 1$

$f'(x) = \frac{1}{3}(1+x)^{-2/3}$

$f'(x) = \frac{1}{3(1+x)^{2/3}}$

$f'(0) = \frac{1}{3(1)^{2/3}} = \frac{1}{3}$

$y - 1 = \frac{1}{3}(x-0)$

$y = 1 + \frac{1}{3}x$

$y(0.3) = 1 + \frac{1}{3} \cdot \frac{3}{10} = \boxed{1.1}$

3. $f(2) = 6$ $y - 6 = \frac{1}{2}(x-2)$

$f'(2) = \frac{1}{2}$ $y = 6 + \frac{1}{2}(x-2)$

$y(2.1) = 6 + \frac{1}{2}(2.1 - 2)$

$\boxed{y(2.1) = 6.05}$

4. $m = -\frac{x+1}{y}$ $(3,2)$

$m = -\frac{2+1}{3} = -2$ $y - 2 = -2(x-3)$

$y = 2 - 2(x-3)$

$y(3.1) = 2 - 2(3.1 - 3) = \boxed{1.8}$

5. $f(x) = \frac{x^2}{x^3+1}$ $x = 2$

$f'(x) = \frac{(x^3+1)(2x) - x^2(3x^2)}{(x^3+1)^2} = \frac{2x^4 + 2x - 3x^4}{(x^3+1)^2}$

$f'(x) = \frac{2x - x^4}{(x^3+1)^2}$

$f'(2) = \frac{4 - 16}{(8+1)^2} = -\frac{12}{81}$ $f(2) = \frac{4}{9}$

$y - \frac{4}{9} = -\frac{12}{81}(x-2)$

$y = \frac{4}{9} - \frac{12}{81}x + \frac{24}{81}$

$y = -\frac{12x}{81} + \frac{20}{27}$ $\boxed{D}$

6. $f(x) = \sqrt{x^2+8}$ $f(a) = \sqrt{a^2+8}$

$f'(x) = \frac{1}{2}(x^2+8)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+8}}$

$f'(a) = \frac{a}{\sqrt{a^2+8}}$

$y - \sqrt{a^2+8} = \frac{a}{\sqrt{a^2+8}}(x-a)$

$y = \frac{ax - a^2}{\sqrt{a^2+8}} + \sqrt{a^2+8} \cdot \frac{\sqrt{a^2+8}}{\sqrt{a^2+8}}$

$y = \frac{ax - a^2 + a^2 + 8}{\sqrt{a^2+8}} = \boxed{\frac{ax+8}{\sqrt{a^2+8}}}$ $\boxed{B}$

7. $f(x) = \frac{1}{2+\sin(\frac{x}{2})} = [2+\sin(\frac{x}{2})]^{-1}$

$f(0) = \frac{1}{2+\sin 0} = \frac{1}{2}$

$f'(x) = -1[2+\sin(\frac{x}{2})]^{-2}(\cos(\frac{x}{2}))(\frac{1}{2})$

$f'(x) = \frac{-\cos(\frac{x}{2})}{2(2+\sin(\frac{x}{2}))^2}$

$f'(0) = \frac{-(1)}{2(2+0)^2} = \frac{-1}{8}$

$y - \frac{1}{2} = \frac{-1}{8}(x-0)$

$y = \frac{-1}{8}x + \frac{1}{2}$

$y(\frac{-1}{10}) = (\frac{1}{8})(\frac{-1}{10}) + \frac{1}{2} = \boxed{\frac{41}{80}}$ C

8. $g(-2) = 6 \quad g'(-2) = -3$

$y - 6 = -3(x+2)$

$y = 6 - 3(x+2)$

$y(-1.97) = 6 - 3(-1.97+2)$

$y(-1.97) \approx \boxed{5.91}$ E

9. $f(1) = -4 \quad f'(1) = 5$

$y + 4 = 5(x-1)$

$y = -4 + 5(x-1)$

$y(1.2) = -4 + 5(1.2-1)$

$\boxed{y(1.2) = -3}$

Since $f''(x) > 0$, f must be CCU

Since $f'(1.2) < 0$, f must be decreasing.

Since f is concave up and decreasing, ↳, the approximation is less than the actual value.

10. a. When $t = 5$, $r = 30$

$V(30) = \frac{4}{3}\pi(30)^3 = 36000\pi$

$V = \frac{4}{3}\pi r^3$

$V' = 4\pi r^2 \frac{dr}{dt}$

$V'(5) = 4\pi(30)^2(2) = 7200\pi$

$y - 36000\pi = 7200\pi(x-30)$

$y = 7200\pi(x-30) + 36000\pi$

When $t = 5.4$