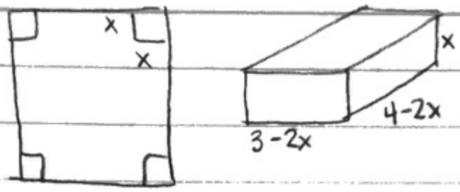
★ Optimization Problems

CP: 

$V = x(3-2x)(4-2x)$

$V = (3x-2x^2)(4-2x)$

$V = 12x - 6x^2 - 8x^2 + 4x$

$V = 4x^3 - 14x^2 + 12x$

$V' = 12x^2 - 28x + 12$

$V' = 4(3x^2 - 7x + 4) = 0$

$x = \frac{7 \pm \sqrt{49-4(3)(4)}}{2(3)}$

$x = \frac{7 \pm \sqrt{13}}{6}$

$x \approx 1.768, \approx 0.566$

$V'' = 4(6x-7)$

$V''(0.566) < 0$

$V''(1.768) > 0$ Also ... not a possible answer.

$\boxed{\therefore V \text{ has a max when } x \approx 0.566}$

1. 1st # $= x$ 2nd # $= y$

$x + 2y = 100$

$x = 100 - 2y$

max $= xy$

$M = (100-2y)y$

$M = 100y - 2y^2$

$M' = 100 - 4y = 0$

when $y = 25$

$M'' = -4$

$M''(25) < 0 \therefore$ a max occurs at $y = 25$

$\boxed{\begin{matrix} y = 25 \\ x = 50 \end{matrix}}$

2. $y = x^2 \quad (0, 6)$

$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$d = \sqrt{(6-x^2)^2 + (0-x)^2}$

$d = \sqrt{36 - 12x^2 + x^4 + x^2}$

$d = \sqrt{x^4 - 11x^2 + 36}$

Let $g(x) = x^4 - 11x^2 + 36$

$g'(x) = 4x^3 - 22x = 0$

$2x(2x^2 - 11) = 0$

$x = 0, \quad 2x^2 - 11 = 0$

$x = \pm\sqrt{11/2}$

$x = \sqrt{\frac{11}{2}} \approx 2.345$

$g''(x) = 12x^2 - 22$

$g''(\sqrt{11/2}) > 0 \therefore x = \sqrt{\frac{11}{2}}$ is a min.

$\boxed{\left(\sqrt{\frac{11}{2}}, \frac{11}{2}\right)}$

3.

$P = 2400\,ft$

$P = 2x + y = 2400$

$y = 2400 - 2x$

$A = x \cdot y$

$A = x(2400 - 2x)$

$A = 2400x - 2x^2$

$A' = 2400 - 4x$

$A' = 0$ when $x = 600$

$A'' = -4x$

$A''(600) = -2400 < 0$

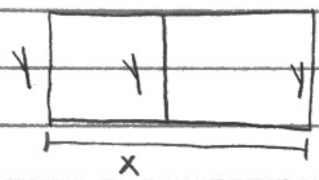
$\therefore$ A has a max at $x = 600$

$x = 600$

$y = 1200$

$\boxed{600ft \times 1200ft}$

4.



$2x + 3y = 102$

$2x = 102 - 3y$

$x = 51 - \frac{3}{2}y$

$A = xy$

$A = (51 - \frac{3}{2}y)y$

$A = 51y - \frac{3}{2}y^2$

$A' = 51 - 3y = 0$

when $y = 17$

$A'' = -3 < 0 \therefore A''(17) < 0$

$\therefore$ A has a max at $y = 17$

If $y = 17$, $x = 25.5$

$A = x \cdot y = \boxed{433.5m^2}$

5.

$V = 20\pi m^3 = \pi r^2 h \Rightarrow 20\pi = \pi r^2 h$

$20 = r^2 h$

$\frac{20}{r^2} = h$

$SA = 2\pi r^2 + 2\pi r h$

$Cost = \$10(2\pi r^2) + \$8(2\pi r h)$

$C = 20\pi r^2 + 16\pi r h$

$C = 20\pi r^2 + 16\pi r\left(\frac{20}{r^2}\right)$

$C = 20\pi r^2 + 320\pi r^{-1}$

$C' = 40\pi r - 320\pi r^{-2}$

$C' = 40\pi r - \frac{320\pi}{r^2}$

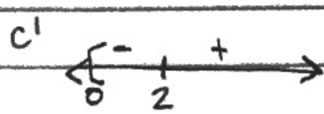
$C' = \frac{40\pi r^3 - 320\pi}{r^2}$

$r = 2,\ h = \frac{20}{4} = 5$

$\boxed{r = 2,\ h = 5}$

$C' = 0$ when $40\pi r^3 = 320\pi$

$r^3 = 8 \Rightarrow r = 2$



$\therefore$ C has a min when $r = 2$