3.6B Curve Sketching

pp. 208-210 (9,11,15,31)

9. $y = \frac{1}{x-2} - 3$

Intercepts: $(0, -3.5)$

$0 = \frac{1}{x-2} - 3$ $\quad (7/3, 0)$

$\frac{3}{1} = \frac{1}{x-2}$ $\quad 3(x-2) = 1$

$x - 2 = \frac{1}{3}$

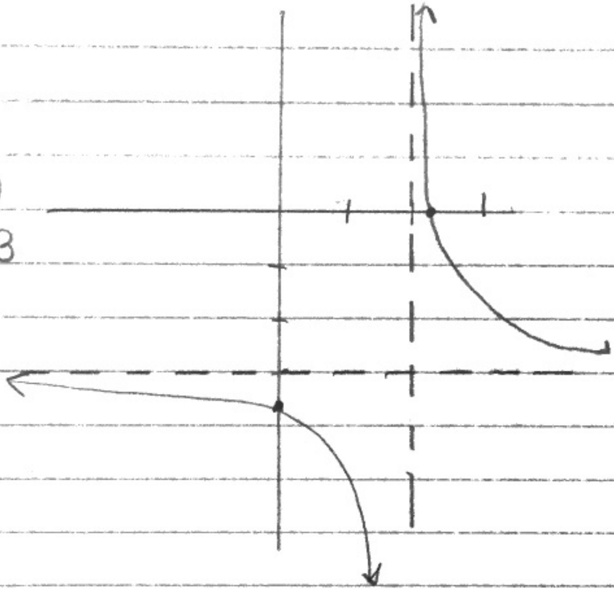
$x = \frac{1}{3} + 2 = \frac{7}{3}$

Intervals	y	y'	y''	Conclusion
$(-\infty, 2)$		−	−	dec, CCD
$x = 2$	∅	∅	∅	VA
$(2, \infty)$		−	+	dec CCU

Asymptotes:

VA @ $x = 2$ (non rem. discont)

HA @ $y = -3$ $\quad \lim_{x \to \infty} \frac{1}{x-2} - 3 = -3$



Extrema: $y = (x-2)^{-1} - 3$

$y' = -1(x-2)^{-2} = \frac{-1}{(x-2)^2}$

y' DNE when x=2, but this is an asymptote

$y'' = 2(x-2)^{-3} = \frac{2}{(x-2)^3}$

y'' DNE when x=2, but this is an asymptote

11. $y = \frac{2x}{x^2-1}$

Intercepts: (0,0)

Asymptotes: x=1, x=-1

$\lim_{x \to \infty} \frac{2x}{x^2-1} = 0$, y=0

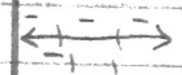
Intervals	y	y'	y''	Conclusions
(-∞,-1)		—	—	dec, CCD
x=-1	∅	∅	∅	Asymptote
(-1,0)		—	+	dec, CCU
x=0	0	—	0	POI, DECREASING
(0,1)		—	—	dec, CCD
x=1	∅	∅	∅	Asymptote
(1,∞)		—	+	dec, CCU

$y' = \frac{(x^2-1)(2)-(2x)(2x)}{(x^2-1)^2}$

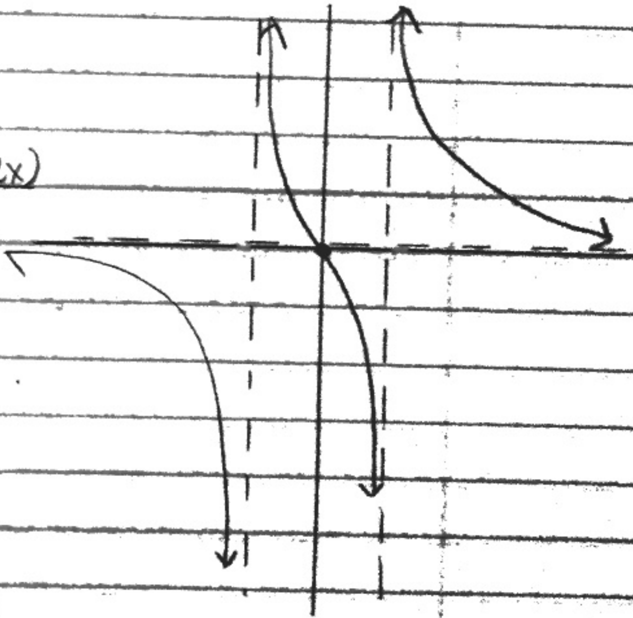
$y' = \frac{2x^2-2-4x^2}{(x^2-1)^2} = \frac{-2-2x^2}{(x^2-1)^2}$

$y' = \frac{-2(x^2+1)}{(x^2-1)^2}$ $y' \neq 0$

y' DNE when



x=±1 (Asymptotes)



$y'' = \frac{(x^2-1)^2(-2)(2x)-(-2)(x^2+1)(2)(x^2-1)(2x)}{(x^2-1)^4}$

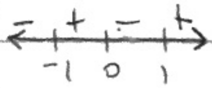
$y'' = \frac{(x^2-1)[-4x(x^2-1)+8x(x^2+1)]}{(x^2-1)^4}$

$y'' = \frac{-4x^3+4x+8x^3+8x}{(x^2-1)^3} = \frac{4x^3+12x}{(x^2-1)^3}$

$y'' = \frac{4x(x^2+3)}{(x^2-1)^3}$ y''=0 when x=0

y'' DNE when x=±1

(Asymptotes)



15. $f(x) = \frac{x^2+1}{x}$

Intercepts: none

Asymptotes: $x=0$

$\lim_{x \to \infty} \frac{x^2+1}{x} = \infty$

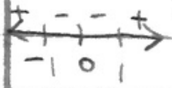
$$\begin{array}{r} x \\ x\,\overline{)\,x^2+1} \\ \underline{x^2} \\ 1 \end{array} \quad \frac{x^2+1}{x} = x + \frac{1}{x}$$

slant asymptote $y=x$

Intervals	f(x)	f'(x)	f''(x)	Conclusion
$(-\infty, -1)$		+	−	Inc, CCD
$x=-1$	−2	0	−	rel max, CCD
$(-1, 0)$		−	−	dec, CCD
$x=0$	∅	∅	∅	Asymptote
$(0, 1)$		−	+	dec, CCU
$x=1$	2	0	+	rel min, CCU
$(1, \infty)$		+	+	inc, CCU

$f'(x) = \frac{x(2x)-(x^2+1)}{x^2} = \frac{2x^2-x^2-1}{x^2}$

$f'(x) = \frac{x^2-1}{x^2}$ $f'(x)=0$ when $x=\pm 1$

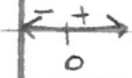


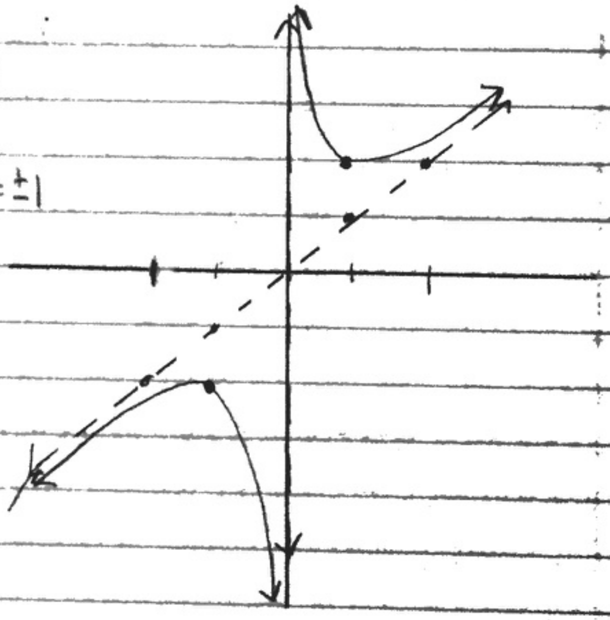
$f'(x)$ DNE @ $x=0$
(Asymptote)

$f''(x) = \frac{(x^2)(2x)-(x^2-1)(2x)}{x^4}$

$f''(x) = \frac{2x^3-2x^3+2x}{x^4} = \frac{2x}{x^4}$

$f''(x) = \frac{2}{x^3}$ $f''(x)$ DNE @ $x=0$





31. $f(x) = 3x^4 + 4x^3$

$f(x) = x^3(3x+4)$

$f(x) = 0$ when $x = 0, \frac{-4}{3}$

$f'(x) = 12x^3 + 12x^2$

$f'(x) = 12x^2(x+1)$

$f'(x) = 0$ when $x = 0, -1$

$f'(x)$ − −1 + 0 +

$f''(x) = 36x^2 + 24x$

$f''(x) = 12x(3x+2)$

$f''(x) = 0$ when $x = 0, -2/3$

\+ −2/3 − 0 +

no asymptotes

Intewals	f(x)	f'(x)	f''(x)	Conclusions
$(-\infty, -1)$		−	+	inc, CCU
$x = -1$	−1	0	+	rel min, CCU
$(-1, -2/3)$		+	+	inc, CCU
$x = -2/3$	$-16/27$	+	0	inc, POI
$(-2/3, 0)$		+	−	inc, CCD
$x = 0$	0	0	0	POI
$(0, \infty$		+	+	inc, CCU