3.6A Curve Sketching

pp. 208-210 (1-5 all, 7, 19, 29)

1. D   2. C   3. A   4. B

5. a. $f'(x) = 0$ when $x = -2, 2$

$f'(x) > 0$ on $(-\infty, -2), (2, \infty)$

$f'(x) < 0$ on $(-2, 2)$

b. $f''(x) = 0$ when $x = 0$

$f''(x) > 0$ on $(0, \infty)$

$f''(x) < 0$ on $(-\infty, 0)$

c. $f'(x)$ increases on $(0, \infty)$ because $f''(x) > 0$ on that interval.

d. $f'(x)$ has a min at $x = 0$ b/c $f''(x)$ changes from positive to negative at $x = 0$.

This means at $x = 0$, the rate of change of f is at its lowest.

Interval	y	y'	y''	Conclusion
$(-\infty, -1)$		−	−	decreasing, ccd
$x = -1$	1/4	−	0	POI
$(-1, 0)$		−	+	decreasing, ccu
$x = 0$	0	0	+	rel. min
$(0, 1)$		+	+	increasing, ccd
$x = 1$	1/4	+	0	POI
$(1, \infty)$		+	−	increasing, ccu

7 $y = \dfrac{x^2}{x^2+3}$

→ Intercepts: (0, 0)

→ No Discontinuities

→ HA: $\lim\limits_{x \to \infty} \dfrac{x^2}{x^2+3} = 1 = \lim\limits_{x \to -\infty} \dfrac{x^2}{x^2+3}$

so HA @ $y = 1$

→ Extrema:

$y' = \dfrac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2} = \dfrac{2x^3+6x-2x^3}{(x^2+3)^2}$

$y' = \dfrac{6x}{(x^2+3)^2}$ $y' = 0$ when $x = 0$

f'(x) − | +
0

y' changes from neg to pos @ $x = 0$, ∴ y must have a relative min @ $x = 0$

→ POI:

$y'' = \dfrac{(x^2+3)^2(6) - (6x)\cdot 2(x^2+3)(2x)}{(x^2+3)^4}$

$y'' = \dfrac{(x^2+3)^2(6) - 24x^2(x^2+3)}{(x^2+3)^4}$

$y'' = \dfrac{(x^2+3)[6(x^2+3) - 24x^2]}{(x^2+3)^4}$

$y'' = \dfrac{6x^2+18-24x^2}{(x^2+3)^3} = \dfrac{18-18x^2}{(x^2+3)^3} = \dfrac{18(1-x^2)}{(x^2+3)^3}$

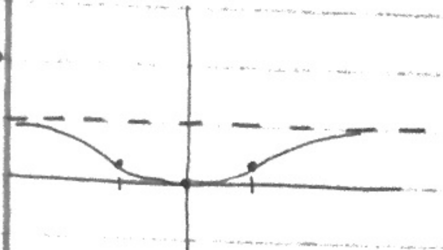
$y'' = \dfrac{18(1-x^2)}{(x^2+3)^3} = 0$ when $x = \pm 1$

− | + | −
-1   1

y'' changes from − to + @ $x = -1$, + to − @ $x = 1$

∴ POI @ $x = \pm 1$

7.



19. $y = x\sqrt{4-x} = x(4-x)^{\frac{1}{2}}$

Domain: $4-x \geq 0$, $\boxed{x \leq 4}$

Intercepts: (0,0) (4,0)

Asymptotes: none

Extrema:

$y' = x \cdot \frac{1}{2}(4-x)^{-1/2}(-1) + (4-x)^{1/2}$

$y' = \frac{-x}{2(4-x)^{+1/2}} + (4-x)^{1/2}$

$y' = \frac{-x + 2(4-x)}{2(4-x)^{1/2}} = \frac{-x+8-2x}{2(4-x)^{1/2}}$

$y' = \frac{8-3x}{2\sqrt{4-x}}$ ∴ $y'=0$ when $\boxed{x = \frac{8}{3}}$

y' DNE when $\boxed{x=4}$

$y'' = \frac{2(4-x)^{1/2}(-3) - (8-3x)(2)(\frac{1}{2})(4-x)^{-1/2}(-1)}{4(4-x)}$

$y'' = \frac{-6(4-x)^{1/2} + \frac{(8-3x)}{(4-x)^{1/2}}}{4(4-x)}$

$y'' = \frac{\frac{-6(4-x) + (8-3x)}{(4-x)^{1/2}}}{4(4-x)}$

$y'' = \frac{-24+6x+8-3x}{(4-x)^{1/2}} \cdot \frac{1}{4(4-x)}$

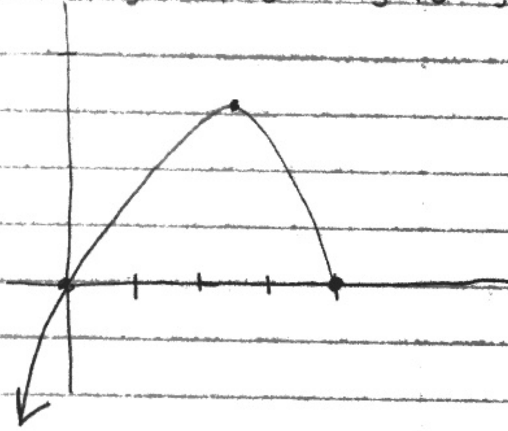
$y'' = \frac{3x-16}{4(4-x)^{3/2}}$ $y''=0$ when $x = 16/3$

but 16/3 not in domain of y

y'' DNE when $x=4$

Interval	y	y'	y''	Conclusion
$(-\infty, 8/3)$		+	−	inc, CCD
$x = 8/3$	$\frac{16}{3\sqrt{3}}$	0	−	rel max, CCD
$(8/3, 4)$		−	−	dec, CCD
$x=4$	0	$\emptyset$	$\emptyset$	endpoint

$y(8/3) = \frac{8}{3}\sqrt{4-\frac{8}{3}} = \frac{8}{3}\sqrt{\frac{12-8}{3}} = \frac{8}{3}\sqrt{\frac{4}{3}} = \frac{16}{3\sqrt{3}} \approx 3.079$



29. $f(x) = 3x^3 - 9x - 1$ Domain $(-\infty, \infty)$

$f(0) = -1$ $(0,-1)$ No Asymptotes

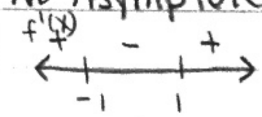
$f'(x) = 9x^2 - 9 = 9(x^2-1)$

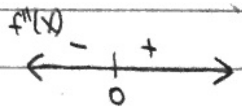
$f'(x) = 9(x-1)(x+1)$

$f'(x) = 0$ when $x = \pm 1$

$f''(x) = 18x$

$f''(x) = 0$ when $x = 0$





Interval	y	y'	y''	Conclusion
$(-\infty,-1)$		+	−	inc, CCD
$x=-1$	5	0	−	rel max, CCD
$(-\infty,0)$		−	−	dec, CCD
$x=0$	−1	−	0	dec, POI
$(0,1)$		−	+	dec, CCU
$x=1$	−7	0	+	rel min, CCU
$(1,\infty)$		+	+	inc, CCU