3.5 Limits at Infinity

pp. 199-201 (1-6 all, 13-27 odds, 49, 53, 61)

1. F
2. C
3. D
4. A
5. B
6. E

17. a. $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^2-4} = 0$

b. $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^{3/2}-4} = -2/3$

c. $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x-4} = -\infty$

13. $f(x) = 5x^3 - 3x^2 + 10$

a. $h(x) = \frac{5x^3-3x^2+10}{x^2}$

$\lim_{x\to\infty} h(x) = \infty$

b. $h(x) = \frac{5x^3-3x^2+10}{x^3}$

$\lim_{x\to\infty} h(x) = 5$

c. $h(x) = \frac{5x^3-3x^2+10x}{x^4}$

$\lim_{x\to\infty} h(x) = 0$

19. $\lim_{x\to\infty} \frac{2x-1}{3x+2} = \boxed{\frac{2}{3}}$

$\lim_{x\to\infty} \frac{\frac{2x}{x}-\frac{1}{x}}{\frac{3x}{x}+\frac{2}{x}} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{3+\frac{2}{x}} = \frac{2}{3}$

21. $\lim_{x\to\infty} \frac{x}{x^2-1} = \boxed{0}$

$\lim_{x\to\infty} \frac{\frac{x}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1-\frac{1}{x^2}} = \frac{0}{1} = 0$

23. $\lim_{x\to-\infty} \frac{5x^2}{x+3} = \boxed{-\infty}$

$\lim_{x\to-\infty} \frac{\frac{5x^2}{x}}{\frac{x}{x}+\frac{3}{x}} = \lim_{x\to-\infty} \frac{5x}{1+\frac{3}{x}} = -\infty$

25. $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2-x}} = \boxed{-1}$

For $x<0$, let $x = -\sqrt{x^2}$

$\lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2-x}}{-\sqrt{x^2}}} = \lim_{x\to\infty} \frac{1}{-\sqrt{1-\frac{1}{x}}} = -\infty$

15. a. $\lim_{x\to\infty} \frac{x^2+2}{x^3-1} = 0$

b. $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = 1$

c. $\lim_{x\to\infty} \frac{x^2+2}{x-1} = \infty$

27. $\lim_{x\to-\infty} \frac{2x+1}{\sqrt{x^2-x}} = \boxed{-2}$

for $x<0$, let $x = -\sqrt{x^2}$

$\lim_{x\to\infty} \frac{\frac{2x}{x}-\frac{1}{x}}{\frac{\sqrt{x^2-x}}{-\sqrt{x^2}}} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{-\sqrt{1-\frac{1}{x}}} = \boxed{-2}$

49. $y = \frac{2+x}{1-x}$

$y' = \frac{(1-x)-(2+x)(-1)}{(1-x)^2} = \frac{1-x+2+x}{(1-x)^2} = \frac{3}{(1-x)^2}$

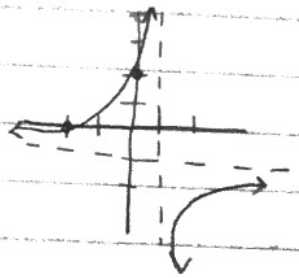
→ $y=0$ when $2+x=0$, $x=-2$

→ y DNE when $1-x=0$, $x=1$ (non removable discont.)

$x=1$ is an asymptote

→ when $x=0$, $y=2$

$y' = \frac{3}{(1-x)^2}$ no relative extrema



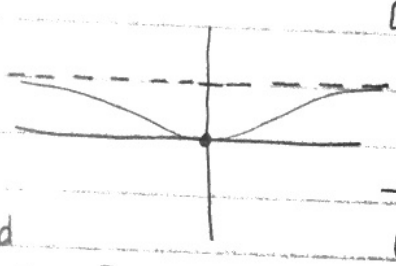
→ $\lim_{x\to\infty} \frac{2+x}{1-x} = -1$

$\lim_{x\to-\infty} \frac{2+x}{1-x} = -1$

Intercepts $(-2,0)$ $(0,2)$

vertical asymptote at $x=1$

53. $y = \frac{x^2}{x^2+9}$



$y=0$ when $x=0$

→ y is never undefined

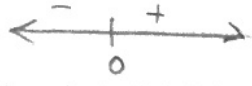
→ $\lim_{x\to\infty} \frac{x^2}{x^2+9} = 1 = \lim_{x\to-\infty} \frac{x^2}{x^2+9}$

$y=1$ is horizontal asymptote

→ $y' = \frac{(x^2+9)(2x)-x^2(2x)}{(x^2+9)^2}$

$y' = \frac{2x^3+18x-2x^2}{(x^2+9)^2} = \frac{18x}{(x^2+9)^2}$

$y'=0$ when $x=0$



$f'(x)$ changes from neg to pos at $x=0$, therefore $f(x)$ has a rel. min @ $x=0$

alt. $y = 2 - \frac{3}{x^2} = 2 - 3x^{-2}$

$y = \frac{2x^2-3}{x^2}$

→ $y=0$ when $2x^2-3=0$ $x = \pm\sqrt{\frac{3}{2}}$

$(\sqrt{\frac{3}{2}},0)$ $(-\sqrt{\frac{3}{2}},0)$

→ $\lim_{x\to\infty} 2 - \frac{3}{x^2} = 2 = \lim_{x\to-\infty} 2 - \frac{3}{x^2}$

hort asymptote @ $y=2$

→ nonremovable discontinuity @ $x=0$ vAsymptote at $x=0$