3.4B 2nd Derivative Test

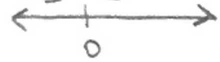
pp. 189-191 (27-39 odds (omit 33), 55, 69)

27. $f(x) = x^4 - 4x^3 + 2$
$f'(x) = 4x^3 - 12x^2$
$f''(x) = 12x^2 - 24x$

$f'(x) = 4x^2(x-3)$
$f'(x) = 0$ when $x = 0, 3$

$f''(0) = 0$ ∴ 2nd DT does not apply

for $x = 0$:



$f'(x)$ does not change sign, so there is no relative extremum at $x=0$

$f''(3) > 0$
∴ $(3, -25)$ is a relative min

29. $f(x) = (x-5)^2$
$f'(x) = 2(x-5)$
$f''(x) = 2$

$f'(x) = 0$ when $x = 5$

$f''(5) > 0$
∴ f has a relative minimum at $(5, 0)$

31. $f(x) = x^3 - 3x^2 + 3$
$f'(x) = 3x^2 - 6x$
$f'(x) = 3x(x-2)$
$f'(x) = 0$ when $x=0, x=2$

$f''(x) = 6x - 6$
$f''(x) = 6(x-1)$
$f''(0) < 0$ ∴ rel max on f at $(0, 3)$
$f''(2) > 0$ ∴ rel min on f at $(2, -1)$

33. $g(x) = x^2(6-x)^3$
$g'(x) = x^2 \cdot 3(6-x)^2(-1) + (6-x)^3(2x)$
$g'(x) = -3x^2(6-x)^2 + 2x(6-x)^3$
$g'(x) = x(6-x)^2[-3x + 2(6-x)]$
$g'(x) = x(6-x)^2(-3x + 12 - 2x)$
$g'(x) = x(6-x)^2(12-5x)$
$g'(x) = 0$ when $x = 6$
$x = \frac{12}{5}$

$g'(x) = (12x - 5x^2)(6-x)^2$
$g''(x) = (12x-5x^2)(2)(6-x)(-1) + (6-x)^2(12-10x)$
$g''(x) = (6-x)[-2(12x-5x^2) + (6-x)(12-10x)]$
$g''(x) = (6-x)[-24x + 10x^2 + 72 - 60x - 12x + 10x^2]$
$g''(x) = (6-x)(20x^2 - 96x + 72)$
$g''(x) = (6-x)4(5x^2 - 24x + 18)$
$g''(x) = 4(6-x)(5x^2 - 24x + 18)$
$g''(0) > 0$ ∴ f has a rel min @ $(0, 0)$
$g''(6) = 0$ test fails → see next page
$g''(12/5) < 0$ ∴ f has a rel max @ $(\frac{12}{5}, 268.7)$

33. cont'd

$g'(x) = (12x - 5x^2)(6-x)^2$

$g'(6) = 0 \quad g''(6) = 0$

so 2nd deriv. test failed

<u>1st Derivative Test</u>

$g'(x)$ does not change sign around $x = 6$

$\therefore$ there is no relative extremum on f at $(6, 0)$

35. $f(x) = x^{2/3} - 3$

$f'(x) = 2/3\, x^{-1/3} = \dfrac{2}{3x^{1/3}}$

$f'(x)$ DNE when $x = 0$

$f''(x) = -2/9\, x^{-4/3} = \dfrac{-2}{9x^{4/3}}$

$f''(0)$ DNE so 2nd DT fails

must use 1st DT

<u>1st DT</u>

$f'(x) = \dfrac{2}{3x^{1/3}}$

f(x): $-$ | $+$ at 0

$\boxed{\text{f'(x) changes sign from neg. to pos. at } x=0 \text{, so f has a relative minimum at } x=0 \; (0,-3)}$

37. $f(x) = x + 4x^{-1}$

$f'(x) = 1 - 4x^{-2}$

$f'(x) = 1 - \dfrac{4}{x^2}$

$f'(x) = \dfrac{x^2 - 4}{x^2}$

$f'(x)$ DNE when $x = 0$

$f'(x) = 0$ when $x = \pm 2$

$f''(x) = 0 + 8x^{-3}$

$f''(x) = \dfrac{8}{x^3}$

$\boxed{\begin{array}{l} f''(0) \text{ DNE but } x=0 \text{ not in domain of } f \\ f''(2) = 1 > 0 \therefore f(x) \text{ has a min at } (2,4) \\ f''(-2) = -1 < 0 \therefore f(x) \text{ has a max at } (-2,-4) \end{array}}$

39. $f(x) = \cos x - x \quad [0, 4\pi]$

$f'(x) = -\sin x - 1$

$f'(x)$ is always less than or equal to zero!

$\therefore$ $f'(x)$ never changes sign and f has no relative extrema

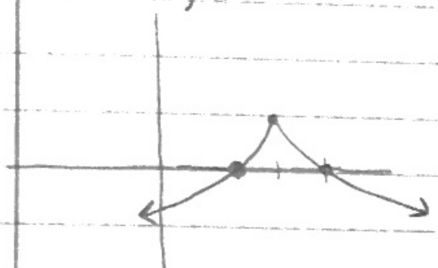
55. $f(2) = f(4) = 0$

$f'(x) > 0$ when $x > 3$

$f'(3)$ DNE

$f'(x) < 0$ if $x > 3$

$f''(x) > 0, \; x \neq 3$



69. $S = \dfrac{5000t^2}{8+t^2}$? time when sales are increasing at GREATEST RATE (maximum of 1st derivative)

$$\frac{dS}{dt} = \frac{(8+t^2)(10000t) - (5000t^2)(2t)}{(8+t^2)^2}$$

$$\frac{ds}{dt} = \frac{80{,}000t + 10000t^3 - 10000t^3}{(8+t^2)^2}$$

$$\frac{dS}{dt} = \frac{80{,}000t}{(8+t^2)^2}$$

$$\frac{d^2S}{dt^2} = \frac{(8+t^2)^2(80000) - (80000t)(2)(8+t^2)(2t)}{(8+t^2)^4}$$

$$\frac{d^2s}{dt^2} = \frac{80{,}000(8+t^2)[8+t^2-4t^2]}{(8+t^2)^4}$$

$$\frac{d^2S}{dt^2} = \frac{80{,}000[8-3t^2]}{(8+t^2)^3}$$

$\dfrac{d^2S}{dt^2} = 0$ when $8 - 3t^2 = 0$

$t^2 = \frac{8}{3} \quad t = \sqrt{\frac{8}{3}} \approx 1.633$

Sales are increasing at the greatest rate at $t = 1.633$ years.