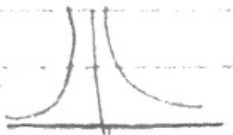
3.3 Increasing and Decreasing Functions and the First Derivative Test

pp. 181-183 (3,5,13,15,19,23,27,29, 35,43,45)

3. increasing $(-\infty, -2)(2, \infty)$
decreasing $(-2, 2)$

5. $f(x) = \frac{1}{x^2}$

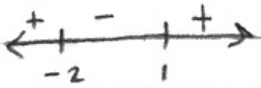


increasing $(-\infty, 0)$
decreasing $(0, \infty)$

13. $f(x) = -2x^2 + 4x + 3$
$f'(x) = -4x + 4$
$0 = -4x + 4 = -4(x-1)$
$\boxed{x = 1}$

increasing $(-\infty, 1)$
decreasing $(1, \infty)$
relative max @ $(1, 5)$

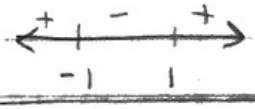
15. $f(x) = 2x^3 + 3x^2 - 12x$
$f'(x) = 6x^2 + 6x - 12$
$0 = 6x^2 + 6x - 12$
$0 = x^2 + x - 2$
$0 = (x+2)(x-1)$
$\boxed{x = -2, 1}$



increasing $(-\infty, -2)(1, \infty)$
decreasing $(-2, 1)$
relative max @ $(-2, 20)$
relative min @ $(1, -7)$

19. $f(x) = \frac{x^5 - 5x}{5} = \frac{1}{5}(x^5 - 5x)$

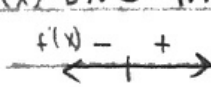
$f'(x) = \frac{1}{5}(5x^4 - 5) = x^4 - 1$
$f'(x) = (x^2+1)(x^2-1)$
$f'(x) = (x^2+1)(x+1)(x-1)$
$0 = (x^2+1)(x+1)(x-1)$
$\boxed{x = 1, -1}$



increasing $(-\infty, -1)(1, \infty)$
decreasing $(-1, 1)$
rel. max @ $(-1, 4/5)$ min @ $(1, -4/5)$

23. $f(x) = (x-1)^{2/3}$
$f'(x) = \frac{2}{3}(x-1)^{-1/3} = \frac{2}{3(x-1)^{1/3}}$

$f'(x)$ DNE when $\boxed{x = 1}$



increasing $(1, \infty)$
decreasing $(-\infty, 1)$
relative max at $(1, 0)$

27. $f(x) = x + \frac{1}{x} = x + x^{-1}$
$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$
$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$
$f'(x) = \frac{(x+1)(x-1)}{x^2}$

$f'(x) = 0$ when $x = \pm 1$
$f'(x)$ DNE when $x = 0$

increasing $(-\infty, -1)(1, \infty)$
decreasing $(-1, 0)(0, 1)$
relative max @ $(-1, -2)$
relative min @ $(1, 2)$

29. $f(x) = \frac{x^2}{x^2-9}$

$f'(x) = \frac{(x^2-9)(2x) - x^2(2x)}{(x^2-9)^2}$

$f'(x) = \frac{2x^3 - 18x - 2x^3}{(x^2-9)^2}$

$f'(x) = \frac{-18x}{(x^2-9)^2} = \frac{-18x}{(x+3)(x-3)}$

$f'(x) = 0$ when $\boxed{x=0}$

$f'(x)$ DNE when $\boxed{x = 3, -3}$

$f'(x)$: $+$ | $+$ | $-$ | $-$ (at -3, 0, 3)

$f(x)$ increases on $(-\infty, -3)(-3, 0)$

decreases on $(0,3)(3,\infty)$

relative maximum at $(0,0)$

33. $f(x) = \frac{x}{2} + \cos x$

$f'(x) = \frac{1}{2} - \sin x$

$0 = \frac{1}{2} - \sin x$

$\sin x = \frac{1}{2}$

$\boxed{x = \pi/6, 5\pi/6}$

$-$ | $+$ | $-$ (at $\pi/6$, $5\pi/6$)

decreasing $(0, \pi/6)(5\pi/6, 2\pi)$

increasing $(\pi/6, 5\pi/6)$

relative max @ $(5\pi/6, \frac{5\pi}{12} - \frac{\sqrt{3}}{2})$

relative min @ $(\pi/6, \frac{\pi}{12} + \frac{\sqrt{3}}{2})$

$f(5\pi/6) = \frac{5\pi}{6} + \cos\frac{5\pi}{6} = \frac{5\pi}{12} - \frac{\sqrt{3}}{2}$

$f(\pi/6) = \frac{\pi/6}{2} + \cos(\pi/6) = \frac{\pi}{12} + \frac{\sqrt{3}}{2}$

35. $f(x) = \sin^2 x + \sin x$

$f'(x) = 2\sin x \cos x + \cos x$

$f'(x) = \cos x (2\sin x + 1)$

$0 = \cos x (2\sin x + 1)$

$\cos x = 0$ $\quad 2\sin x + 1 = 0$

$\boxed{x = \pi/2, 3\pi/2}$ $\quad \sin x = -\frac{1}{2}$

$\boxed{x = 7\pi/6, 11\pi/6}$

$+$ | $-$ | $+$ | $-$ | $+$ (at $\frac{\pi}{2}$, $7\pi/6$, $3\pi/2$, $11\pi/6$)

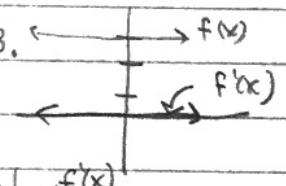
increasing on $(0, \pi/2)(\frac{7\pi}{6}, \frac{3\pi}{2})$ $(11\pi/6, 2\pi)$

rel. max @ $(\frac{\pi}{2}, 2)(\frac{3\pi}{2}, 0)$

rel. min @ $(\frac{7\pi}{6}, -\frac{1}{4})(\frac{11\pi}{6}, -\frac{1}{4})$

decreasing on $(\pi/2, 7\pi/6)(3\pi/2, 11\pi/6)$

43.



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