★ 3.2 Rolle's Thm and the Mean Value Theorem

pp. 172-173 (1-19 odds, 25, 31, 33, 35, 37, 41, 43)

1. $f(x)$ is not differentiable on $(0,2)$

3. $f(x) = x^2 - x - 2$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2, -1$

$f'(x) = 2x - 1$

$2x - 1 = 0$ when $x = \frac{1}{2}$

$\boxed{(2,0) + (-1,0) \quad f'(\tfrac{1}{2}) = 0}$

5. $f(x) = x\sqrt{x+4}$

$0 = x\sqrt{x+4}$

$x = 0, -4$

$f'(x) = x \cdot \frac{1}{2}(x+4)^{-1/2} + (x+4)^{1/2}$

$f'(x) = \frac{x}{2(x+4)^{1/2}} + (x+4)^{1/2}\frac{2(x+4)^{1/2}}{2(x+4)^{1/2}}$

$f'(x) = \frac{x + 2(x+4)}{2(x+4)^{1/2}} = \frac{3x+8}{2\sqrt{x+4}}$

$f'(x) = 0$ when $3x + 8 = 0$

$x = -8/3$

$\boxed{(0,0) \quad (-4,0) \quad f'(-8/3) = 0}$

7. $f(x) = x^2 - 2x \quad [0,2]$

$f(0) = 0 \quad f(2) = 0$

f is continuous on $[0,2]$
f is diff on $(0,2)$
$f(0) = f(2) = 0$ ∴ Rolle's Thm applies

$f'(x) = 2x - 2, \quad 0 = 2x - 2, \quad x = 1$

$\boxed{c = 1}$

9. $f(x) = (x-1)(x-2)(x-3), \ [1,3]$

f is continuous on $[-1,3]$
f is diff on $(-1,3)$
$f(-1) = f(3) = 0$

$f(x) = (x-1)(x-2)(x-3)$

$f(x) = (x^2 - 2x - x + 2)(x-3)$

$f(x) = (x^2 - 3x + 2)(x-3)$

$f(x) = x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$

$f(x) = x^3 - 6x^2 + 11x - 6$

$f'(x) = 3x^2 - 12x + 11$

$3x^2 - 12x + 11 = 0$

$x = \frac{12 \pm \sqrt{144 - 4(3)(11)}}{2(3)} = \frac{12 \pm \sqrt{12}}{6}$

$x = \frac{12 \pm \sqrt{4}\sqrt{3}}{6} = \frac{12 \pm 2\sqrt{3}}{6} = \frac{6 \pm \sqrt{3}}{3}$

$\boxed{c = \frac{6+\sqrt{3}}{3} \text{ and } \frac{6-\sqrt{3}}{3}}$

11. $f(x) = x^{2/3} - 1 \quad [-8,8]$

$f(x)$ is continuous on $[-8,8]$
but it is not differentiable
$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$
because $f'(x)$ DNE when $x = 0$
∴ Rolle's Thm does not apply

13. $f(x) = \frac{x^2-2x-3}{x+2}$ $[-1,3]$

$f(x) = \frac{(x-3)(x+1)}{(x+2)}$

$f(-1) = f(3) = 0$
$f(x)$ is continuous on $[-1,3]$
$f(x)$ is differentiable on $(-1,3)$
∴ Rolle's Thm Applies

$f'(x) = \frac{(x+2)(2x-2)-(x^2-2x-3)}{(x+2)^2}$

$f'(x) = \frac{2x^2-2x+4x-4-x^2+2x+3}{(x+2)^2}$

$f'(x) = \frac{x^2+4x-1}{(x-2)^2}$

$f'(x)=0$ when $x^2+4x-1=0$

$x = \frac{-4 \pm \sqrt{16-4(1)(-1)}}{2(1)} = \frac{-4 \pm \sqrt{20}}{2}$

$x = \frac{-4 \pm \sqrt{4}\cdot\sqrt{5}}{2} = \frac{-4 \pm 2\sqrt{5}}{2}$

$x = -2 \pm \sqrt{5}$

$\boxed{c = -2+\sqrt{5}}$

* $-2-\sqrt{5}$ is not in original interval $[-1,3]$

15. $f(x) = \sin x$ $[0, 2\pi]$

$f(0) = f(2\pi) = 0$
f is continuous on $[0,2\pi]$
f is differentiable on $(0,2\pi)$
∴ Rolle's Thm Applies

$f'(x) = \cos x$

$\cos x = 0$ when $x = \pi/2, 3\pi/2$

$\boxed{c = \pi/2, 3\pi/2}$

19. $f(x) = \tan x$ $[0,\pi]$

$f(0) = f(\pi) = 0$
$f(x)$ is not continuous on $[0,\pi]$
∴ Rolle's Thm does not apply

25. $f(t) = -16t^2+48t+32$

a. $f(1) = 32 = f(2)$

b. the velocity must be 0 at some time on $[1,2]$

$f'(t) = -32t+48$

$0 = -32t+48$

$t = \frac{48}{32} = \boxed{1.5 \text{ sec}}$

31. $f(x) = x^2$ $[-2,1]$

$f(x)$ is continuous on $[-2,1]$ and diff on $(-2,1)$.

$\frac{f(1)-f(-2)}{1-{-2}} = \frac{1-4}{3} = \frac{-3}{3} = -1$

$f'(x) = 2x$

$2x = -1$ when $x = -\frac{1}{2}$

$\boxed{c = -\frac{1}{2}}$

33. $f(x) = x^{2/3}$ $[0,1]$

$f(x)$ is continuous on $[0,1]$ and differentiable on $(0,1)$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

$\frac{f(1)-f(0)}{1-0} = \frac{1-0}{1} = 1$

$\frac{2}{3x^{1/3}} = 1$

$3x^{1/3} = 2$

$x^{1/3} = \frac{2}{3}$

$x = (\frac{2}{3})^3 = \frac{8}{27}$

$\boxed{c = \frac{8}{27}}$

35. $f(x) = \sqrt{2-x}$ $[-7,2]$

$f'(x) = \frac{1}{2}(2-x)^{-1/2}(-1)$

$f'(x) = \frac{-1}{2\sqrt{2-x}}$

$\boxed{f(x) \text{ is continuous on } [-7,2] \text{ and differentiable on } (-7,2)}$

$\frac{f(2)-f(-7)}{2-{}^-7} = \frac{0-3}{9} = \frac{-1}{3}$

$f'(x) = \frac{-1}{2\sqrt{2-x}} = \frac{-1}{3}$

$2\sqrt{2-x} = 3$

$\sqrt{2-x} = \frac{3}{2}$

$2-x = \frac{9}{4}$

$x = {}^-\frac{1}{4}$

$\boxed{c = -1/4}$

37. $f(x) = \sin x$ $[0,\pi]$

$f'(x) = \cos x$

$\boxed{f(x) \text{ is continuous on } [0,\pi] \text{ and differentiable on } (0,\pi)}$

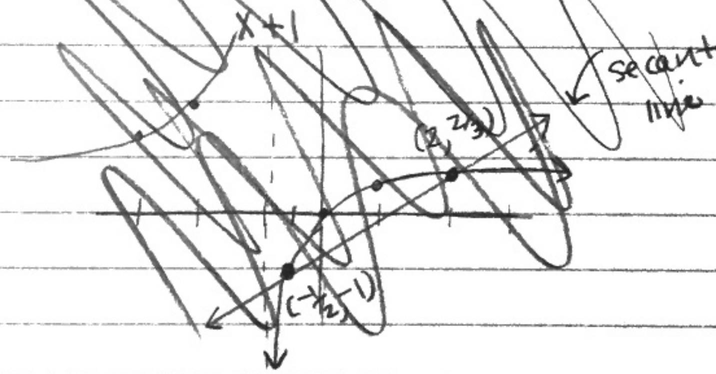
$\frac{f(\pi)-f(0)}{\pi-0} = \frac{0-0}{\pi-0} = \frac{0}{\pi} = 0$

$f'(x) = \cos x = 0$

$x = \pi/2$

$\boxed{c = \pi/2}$

39. $f(x) = \frac{x}{x+1}$ $[-\frac{1}{2}, 2]$



secant line: $m = \frac{\frac{2}{3} - {}^-1}{2 - {}^-\frac{1}{2}} = \frac{5/3}{5/2} = \frac{2}{3}$

$\boxed{y - \frac{2}{3} = \frac{2}{3}(x-2) \text{ or } y+1 = \frac{2}{3}(x+\frac{1}{2})}$

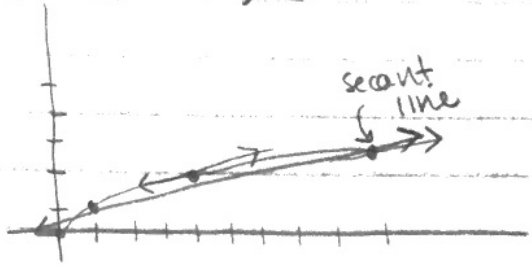
$f'(x) = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2},\ \frac{1}{(x+1)^2} = \frac{2}{3}$

$(x+1)^2 = \frac{3}{2}$

$x+1 = \pm\sqrt{\frac{3}{2}}$

$x = -1 \pm \sqrt{\frac{3}{2}}$

41. $f(x)=\sqrt{x}$ $[1,9]$



$f(1)=1$ $f(9)=3$

secant line : $m=\frac{3-1}{9-1}=\frac{2}{8}=\frac{1}{4}$

$\boxed{y-3=\frac{1}{4}(x-9) \text{ or } y-1=\frac{1}{4}(x-1)}$

tangent line: $f'(x)=\frac{1}{2}x^{-1/2}$

$f'(x)=\frac{1}{2\sqrt{x}}=\frac{1}{4}$

$2\sqrt{x}=4,\ \sqrt{x}=2$

$x=4$

when $x=4,\ y=2$

$\boxed{y-2=\frac{1}{4}(x-4)}$

43. $s(t)=-4.9t^2+500$

A. $[0,3]$

average velocity

$\frac{s(3)-s(0)}{3-0}=\frac{455.9-500}{3}=\boxed{-14.7 \text{ m/sec}}$

B. $s'(t)=-9.8t$ $\quad -9.8t=-14.7$

$s'(t)=-14.7$ $\quad t=1.5$

$\boxed{1.5 \text{ seconds}}$