

2.6A pp. 149-150 (1, 3, 5, 13, 19, 23, 27, 29, 33)

1. $y = \sqrt{x} \quad \frac{dy}{dt} = \frac{1}{2} x^{-1/2} \frac{dx}{dt}$

A. $x = 4 \quad \frac{dx}{dt} = 3$

$\frac{dy}{dt} = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} \cdot 3 = \boxed{\frac{3}{4}}$

B. $x = 25 \quad \frac{dy}{dt} = 2$

$2 = \frac{1}{2} \cdot \frac{1}{\sqrt{25}} \cdot \frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = 20}$

3. $xy = 4 \quad x \frac{dy}{dt} + y \frac{dx}{dt} = 0$

A. $x = 8 \quad \frac{dx}{dt} = 10 \quad y = \frac{1}{2}$

$8 \frac{dy}{dt} + \frac{1}{2} (10) = 0$

$8 \frac{dy}{dt} = -5$

$\boxed{\frac{dy}{dt} = -\frac{5}{8}}$

B. $x = 1 \quad \frac{dy}{dt} = -6 \quad y = 4$

$(1)(-6) + 4(\frac{dx}{dt}) = 0$

$-6 + 4 \frac{dx}{dt} = 0$

$\boxed{\frac{dx}{dt} = \frac{3}{2}}$

5. $y = x^2 + 1 \quad \frac{dx}{dt} = 2$

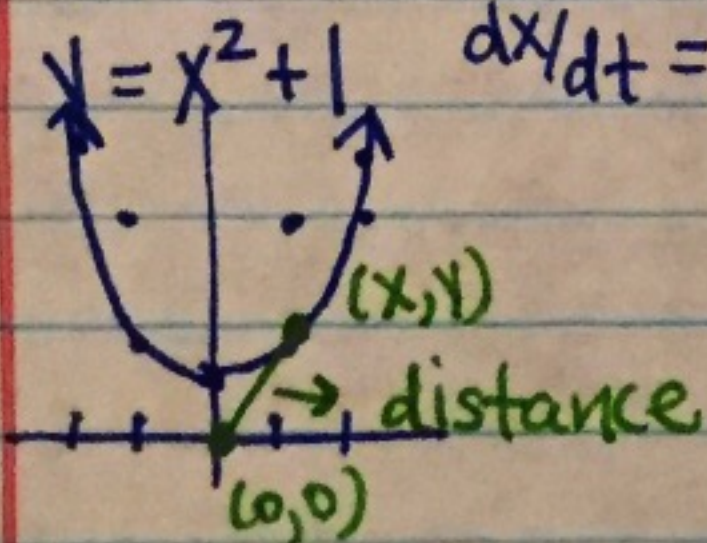
$\frac{dy}{dt} = 2x \frac{dx}{dt}$

A. $x = -1 \quad \left. \frac{dy}{dt} \right|_{x=-1} = 2(-1)(2) = \boxed{-4}$

B. $x = 0 \quad \left. \frac{dy}{dt} \right|_{x=0} = 2(0)(2) = \boxed{0}$

C. $x = 1 \quad \left. \frac{dy}{dt} \right|_{x=1} = 2(1)(2) = \boxed{4}$

13. $y = x^2 + 1 \quad \frac{dx}{dt} = 2$



$Dd = \sqrt{(y-0)^2 + (x-0)^2}$

$Dd = \sqrt{y^2 + x^2}$

$Dd = \sqrt{(x^2+1)^2 + x^2}$

$Dd = \sqrt{x^4 + 2x^2 + 1 + x^2}$

$D = \sqrt{x^4 + 3x^2 + 1} = (x^4 + 3x^2 + 1)^{1/2}$

$\frac{dD}{dt} = \frac{1}{2} (x^4 + 3x^2 + 1)^{-1/2} (4x^3 + 6x) \left(\frac{dx}{dt} \right)$

$\frac{dD}{dt} = \frac{4x^3 + 6x}{2\sqrt{x^4 + 3x^2 + 1}} (2) = \boxed{\frac{4x^3 + 6x}{\sqrt{x^4 + 3x^2 + 1}}}$

19. $\frac{dV}{dt} = 800 \frac{\text{cm}^3}{\text{min}}$? $\frac{dr}{dt}$ A. $r=30$ B. $r=60$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$800 = 4\pi (30)^2 \left(\frac{dr}{dt}\right)$

$800 = 4\pi (60)^2 \frac{dr}{dt}$

$\frac{2}{9\pi} = \frac{dr}{dt}$

$\frac{1}{18\pi} = \frac{dr}{dt}$

22. $V = \frac{1}{3}\pi r^2 h$ A. $r=6$ B. $r=24$

$\frac{dr}{dt} = 2$

$h = 3r$

$V = \frac{1}{3}\pi r^2 (3r)$

$V = \pi r^3$

$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 3\pi (6)^2 (2)$

$\frac{dV}{dt} = 3\pi (24)^2 (2)$

$\frac{dV}{dt} = 216\pi \text{ in}^3/\text{min}$

$\frac{dV}{dt} = 3456\pi \text{ in}^3/\text{min}$

23. $V = \frac{1}{3}\pi r^2 h$ $d = 3h$ $V = \frac{1}{3}\pi \left[\frac{3}{2}h\right]^2 h$

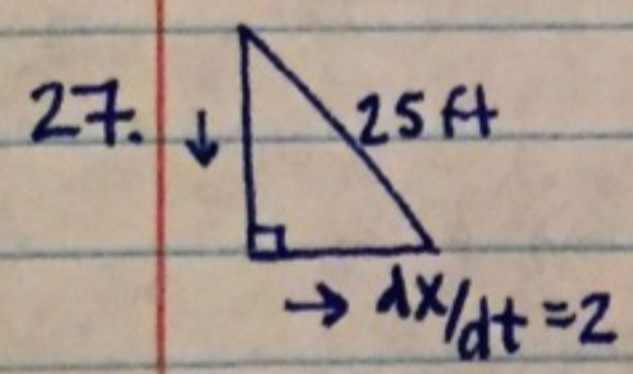
$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$ $2r = 3h$ $V = \frac{3}{4}\pi h^3$

$d = 3h$ $h = 15$ $r = \frac{3}{2}h$ $\frac{dV}{dt} = \frac{9}{4}\pi h^2 \cdot \frac{dh}{dt}$

? $\frac{dh}{dt}$

$10 = \frac{9}{4}\pi (15)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{405\pi} \text{ ft}/\text{min}$



A. ? $\frac{dy}{dt}$

$x^2 + y^2 = 25$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$

If $x=7, y=24$ $7(2) + 24\left(\frac{dy}{dt}\right) = 0$

If $x=15, y=20$ $15(2) + 20\left(\frac{dy}{dt}\right) = 0$

$\frac{dy}{dt} = \frac{-7}{12}$

$\frac{dy}{dt} = \frac{-3}{2}$

If $x=24, y=7$

$24(2) + 7 \frac{dy}{dt} = 0$

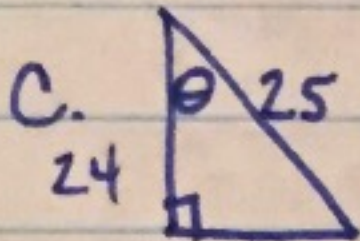
$\frac{dy}{dt} = \frac{-48}{7}$

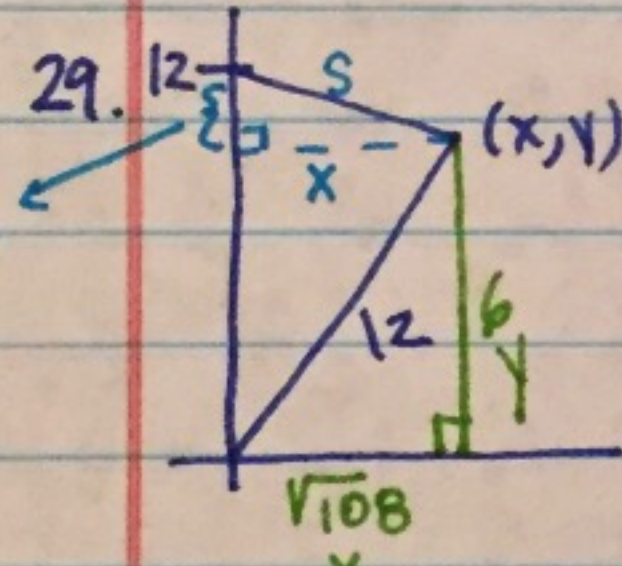
27. B. $A = \frac{xy}{2} = \frac{1}{2}xy$ $\frac{dx}{dt} = 2$ $x = 7$ $\frac{dy}{dt} = \frac{-7}{12}$
 $y = 24$

$$\frac{dA}{dt} = \frac{1}{2}x \frac{dy}{dt} + \frac{1}{2}y \frac{dx}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2}(7)\left(\frac{-7}{12}\right) + \frac{1}{2}(24)(2)$$

$$\frac{dA}{dt} = \boxed{\frac{527 \text{ ft}^2}{24 \text{ sec}}} \approx \boxed{21.958 \text{ ft}^2/\text{sec}}$$

28. C.  $\sin \theta = \frac{x}{25}$ $\cos \theta \cdot \frac{d\theta}{dt} = \frac{1}{25} \frac{dx}{dt}$
 $\frac{dx}{dt} = 2$ $x = 7$ $\frac{24}{25} \cdot \frac{d\theta}{dt} = \frac{1}{25}(2)$
 $\cos \theta = \frac{24}{25}$ $\boxed{\frac{d\theta}{dt} = \frac{1}{12} \text{ rad/sec}}$

29.  $\frac{ds}{dt} = -0.2 \text{ m/sec}$ $\frac{dx}{dt} = ?$ $\frac{dy}{dt} = ?$
 $y = 6$

$$x^2 + (12-y)^2 = s^2$$

$$2x \frac{dx}{dt} + 2(12-y) \frac{dy}{dt} = 2s \frac{ds}{dt}$$

~~$$2(\sqrt{108}) \frac{dx}{dt} + 2(12-6) \frac{dy}{dt} = 2(12)(-0.2)$$~~

to find s:
 $x^2 + (12-y)^2 = s^2$
 $(\sqrt{108})^2 + (12-6)^2 = s^2$
 $108 + 36 = s^2$
 $144 = s^2$
 $12 = s$

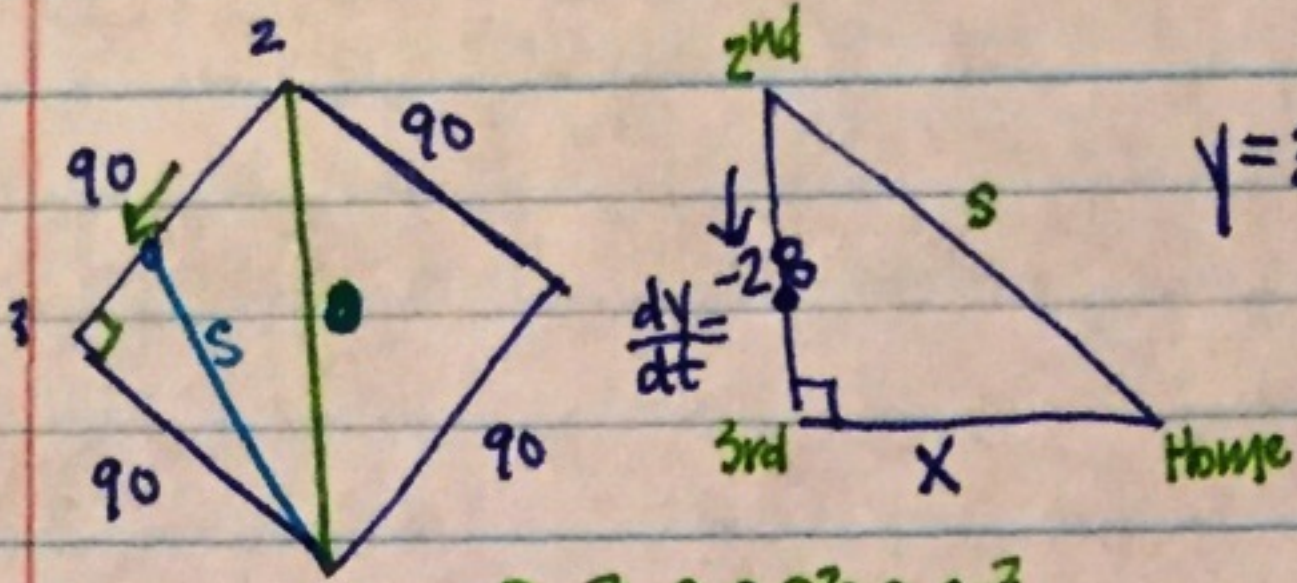
to eliminate $\frac{dy}{dt}$:
 $x^2 + y^2 = 12^2$
 $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$
 $\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$

now substitute:
 $(\sqrt{108}) \frac{dx}{dt} + (12-6) \frac{x}{y} \frac{dx}{dt} = 12(-0.2)$
 $\sqrt{108} \frac{dx}{dt} + 6 \cdot \frac{\sqrt{108}}{6} \cdot \frac{dx}{dt} = -2.4$
 $2\sqrt{108} \frac{dx}{dt} = -2.4$
 $\frac{dx}{dt} \approx -0.115 \text{ m/sec}$

$$\frac{dy}{dt} = \frac{-\sqrt{108}}{6} [-0.1154700538]$$

$$\boxed{\frac{dy}{dt} = -0.2 \text{ m/sec}}$$

33.



$$y = 30 \text{ ft}$$

$$x^2 + y^2 = s^2$$

$$90^2 + y^2 = s^2$$

$$0 + 2y \frac{dy}{dt} = 2s \frac{ds}{dt}$$

$$2(30)(-28) = 2(\sqrt{9000}) \frac{ds}{dt}$$

$$\frac{ds}{dt} = -8.854 \text{ ft/sec}$$

~~$$90^2 + 30^2 = s^2$$~~

$$s^2 = x^2 + y^2$$

$$s^2 = 90^2 + 30^2$$

$$s = \sqrt{9000}$$