2.5 pp. 142-143 (1, 7, 13, 21, 25, 27, 29, 35, 43, 55)

1. $x^2+y^2=36$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\boxed{\frac{dy}{dx}=\frac{-x}{y}}$

7. $x^3y^3-y=x$

$x^3\cdot 3y^2\frac{dy}{dx}-y^3\cdot 3x^2-\frac{dy}{dx}=1$

$3y^2x^3\,dy/dx-dy/dx=1+3x^2y^3$

$\frac{dy}{dx}(3x^3y^2-1)=1+3x^2y^3$

$\boxed{\frac{dy}{dx}=\frac{1+3x^2y^3}{3x^3y^2-1}}$

13. $\sin x=x(1+\tan y)$

$\cos x=x(\sec^2y\,dy/dx)+(1+\tan y)(1)$

$\cos x=x\sec^2y\,dy/dx+1+\tan y$

$-x\sec^2y\,dy/dx=1+\tan y-\cos x$

$\boxed{\frac{dy}{dx}=\frac{-1+\tan y+\cos x}{x\sec^2y}}$

21. $xy=4\quad(-4,-1)$

$x\,dy/dx+y=0$

$x\,dy/dx=-y$

$\frac{dy}{dx}=\frac{-y}{x}$

$\left.\frac{dy}{dx}\right|_{(-4,-1)}=\frac{-(-1)}{-4}=\boxed{\frac{-1}{4}}$

25. $x^{2/3}+y^{2/3}=5\quad(8,1)$

$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$

$\frac{2}{3}y^{-1/3}\frac{dy}{dx}=\frac{-2}{3x^{1/3}}$

$\frac{2}{3y^{1/3}}\frac{dy}{dx}=\frac{-2}{3x^{1/3}}$

$\boxed{\frac{dy}{dx}=\frac{-y^{1/3}}{x^{1/3}}}$

$\left.\frac{dy}{dx}\right|_{(8,1)}=\frac{-\sqrt[3]{1}}{\sqrt[3]{8}}=\boxed{\frac{-1}{2}}$

27. $\tan(x+y)=x\quad(0,0)$

$\sec^2(x+y)(1+dy/dx)=1$

$\sec^2(x+y)+\sec^2(x+y)\,dy/dx=1$

$\sec^2(x+y)\,dy/dx=1-\sec^2(x+y)$

$\frac{dy}{dx}=\frac{1-\sec^2(x+y)}{\sec^2(x+y)}=\frac{-\tan^2(x+y)}{\sec^2(x+y)}$

$\left.\frac{dy}{dx}\right|_{(0,0)}=\frac{1-\sec^2(0)}{\sec^2(0)}=\frac{1-1}{1}=\boxed{0}$

$\boxed{-\sin^2(x+y)}$

29. $(x^2+4)y=8\quad(2,1)$

$(x^2+4)\,dy/dx+y(2x)=0$

$(x^2+4)\,dy/dx=-2xy$

$\frac{dy}{dx}=\frac{-2xy}{x^2+4}$

$\left.\frac{dy}{dx}\right|_{(2,1)}=\frac{-2(2)(1)}{2^2+4}=\frac{-4}{8}=\boxed{\frac{-1}{2}}$

35. $x^2+y^2=36$

$2x+2y\,dy/dx=0$

$\frac{dy}{dx}=\frac{-2x}{2y}=\frac{-x}{y}$

$\frac{d^2y}{dx^2}=\frac{y(-1)-(-x)\,dy/dx}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-y+x\,dy/dx}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-y+x\frac{-x}{y}}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-y-\frac{x^2}{y}}{y^2}$

$\boxed{\frac{d^2y}{dx^2}=\frac{-y^2-x^2}{y^3}}=\boxed{\frac{-36}{y^3}}$ sub HERE

55. $2y^2-3x^4=0$

w/ respect to x

$4y\,dy/dx-12x^3=0$

$4y\,dy/dx=12x^3$

$\frac{dy}{dx}=\frac{12x^3}{4y}=\boxed{\frac{3x^3}{y}}$

w/ respect to y

$4y-12x^3\frac{dx}{dy}=0$

$-12x^3\,dx/dy=-4y$

$\frac{dx}{dy}=\frac{-4y}{-12x^3}=\boxed{\frac{y}{3x^3}}$

43. $x^2+y^2=25$ $(4,3)$ $(-3,4)$

$2x+2y\,dy/dx=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-x}{y}$

$\left.\frac{dy}{dx}\right|_{(4,3)}=\frac{-4}{3}$ $\left.\frac{dy}{dx}\right|_{(-3,4)}=\frac{3}{4}$

| tangent | tangent |
|---|---|
| $y-3=\frac{-4}{3}(x-4)$ | $y-4=\frac{3}{4}(x+3)$ |
| normal | normal |
| $y-3=\frac{3}{4}(x-4)$ | $y-4=\frac{-4}{3}(x+3)$ |