2.1B pp. 104-104 (21, 23, 25, 27, 33, 61, 71, 73, 75, 77)

21. $f(x) = \frac{1}{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)-1} - \frac{1}{x-1}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x-1} - \frac{1}{x-1}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{x-1-(x+\Delta x-1)}{(x+\Delta x-1)(x-1)}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x-1-x-\Delta x+1}{\Delta x(x+\Delta x-1)(x-1)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x+\Delta x-1)(x-1)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x-1)(x-1)}$$

$$\boxed{f'(x) = \frac{-1}{(x-1)^2}}$$

23. $f(x) = \sqrt{x+1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+1} - \sqrt{x+1}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+1} - \sqrt{x+1}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+1} + \sqrt{x+1}}{\sqrt{x+\Delta x+1} + \sqrt{x+1}}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x+1) - (x+1)}{\Delta x[\sqrt{x+\Delta x+1} + \sqrt{x+1}]}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(\sqrt{x+\Delta x+1} + \sqrt{x+1})}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+1} + \sqrt{x+1}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x+1}}}$$

(21, 23, 25, 27, 33, 61, 71, 73, 75, 77)

2.1B pp. 101-104 ~~(25, 27, 33, 35, 61, 71, 73, 75, 77)~~

25. $f(x) = x^2 + 1 \quad (2,5)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 1 - [x^2+1]}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 + 1 - x^2 - 1}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} 2x + \Delta x = 2x$$

$f'(2) = 4$

$$\boxed{y - 5 = 4(x-2)}$$

27. $f(x) = x^3 \quad (2,8)$

$$f'(2) = \lim_{\Delta x \to 0} \frac{(2+\Delta x)^3 - 2^3}{\Delta x}$$

$$f'(2) = \lim_{\Delta x \to 0} \frac{8 + 3(2)^2(\Delta x) + 3(2)\Delta x^2 + \Delta x^3 - 8}{\Delta x}$$

$$f'(2) = \lim_{\Delta x \to 0} \frac{8 + 12\Delta x + 6\Delta x^2 + \Delta x^3 - 8}{\Delta x}$$

$$f'(2) = \lim_{\Delta x \to 0} \frac{12\Delta x + 6\Delta x^2 + \Delta x^3}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} 12 + 6\Delta x + \Delta x^2 = 12$$

$$\boxed{y - 8 = 12(x-2)}$$

33. $f(x) = x^3 \quad 3x - y + 1 = 0$

$y = 3x + 1, \; m = 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - x^3}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x\Delta x^2 + \Delta x^3}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} 3x^2 + 3x\Delta x + \Delta x^2 = 3x^2$$

$f'(x) = 3x^2 \qquad 3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$m = 3$

$(-1, -1) \quad \boxed{y + 1 = 3(x+1)}$

$(1, 1) \quad \boxed{y - 1 = 3(x-1)}$

$$\cancel{f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x}}$$

$$\cancel{f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x}}$$

$$\cancel{f'(x) = \lim_{\Delta x \to 0} \frac{\frac{\sqrt{x} - \sqrt{x+\Delta x}}{\sqrt{x}\cdot\sqrt{x+\Delta x}}}{\Delta x}}$$

$$\cancel{f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x} - \sqrt{x+\Delta x}}{\Delta x\sqrt{x^2 + x\Delta x}}}$$

61. $f(x)=x^2-1 \quad c=2$

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

$$f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$$

$$f'(2)=\lim_{x\to 2}\frac{x^2-1-[4-1]}{x-2}$$

$$f'(2)=\lim_{x\to 2}\frac{x^2-4}{x-2}$$

$$f'(2)=\lim_{x\to 2}\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}=\boxed{4}$$

71. $f(x)$ is differentiable when $x\neq -3$ or $(-\infty,-3)\cup(-3,\infty)$

73. $f(x)$ is differentiable when $x\neq -1$ or $(-\infty,-1)\cup(-1,\infty)$

75. $f(x)$ is differentiable when $x\neq 3$ or $(-\infty,3)\cup(3,\infty)$

77. $f(x)$ is differentiable on $(1,\infty)$